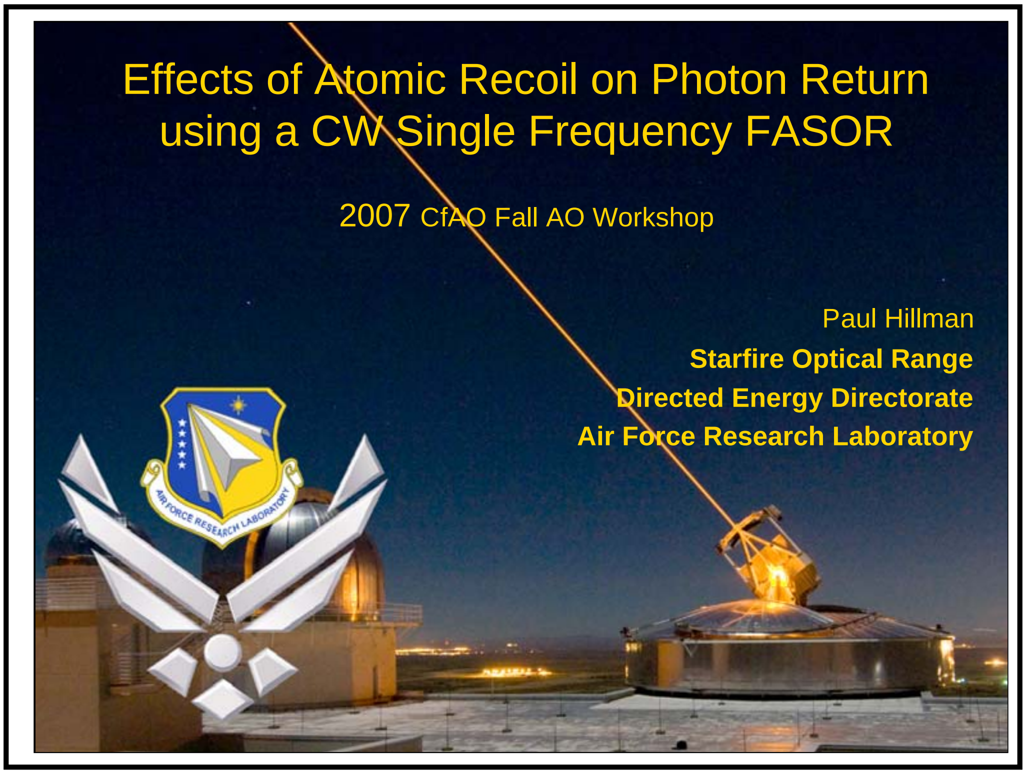


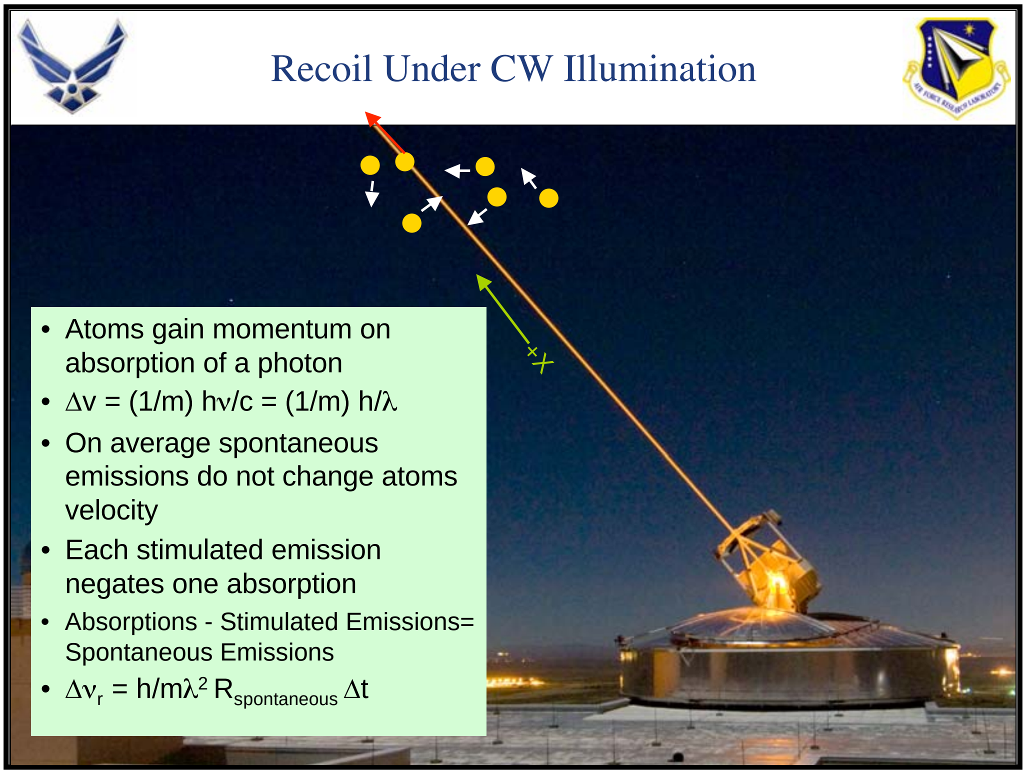
COVER SLIDE

The following slide presentation came out of questions of why we were not seeing much saturation effects. If we'd measure the photon return rates at various Fasor powers, at 40W power and a beam size of 0.7m FWHM in the mesosphere we should be close to the saturation intensity near the center of the beam. Further, Jeys determined a velocity distribution of sodium atoms under CW illumination, but did not calculate how that affected the photon return from a guide star. Milonni uses a complicated model for the sodium interaction and a much simpler model is desired; he determined recoil effects on the return but the work was never published.

Outline

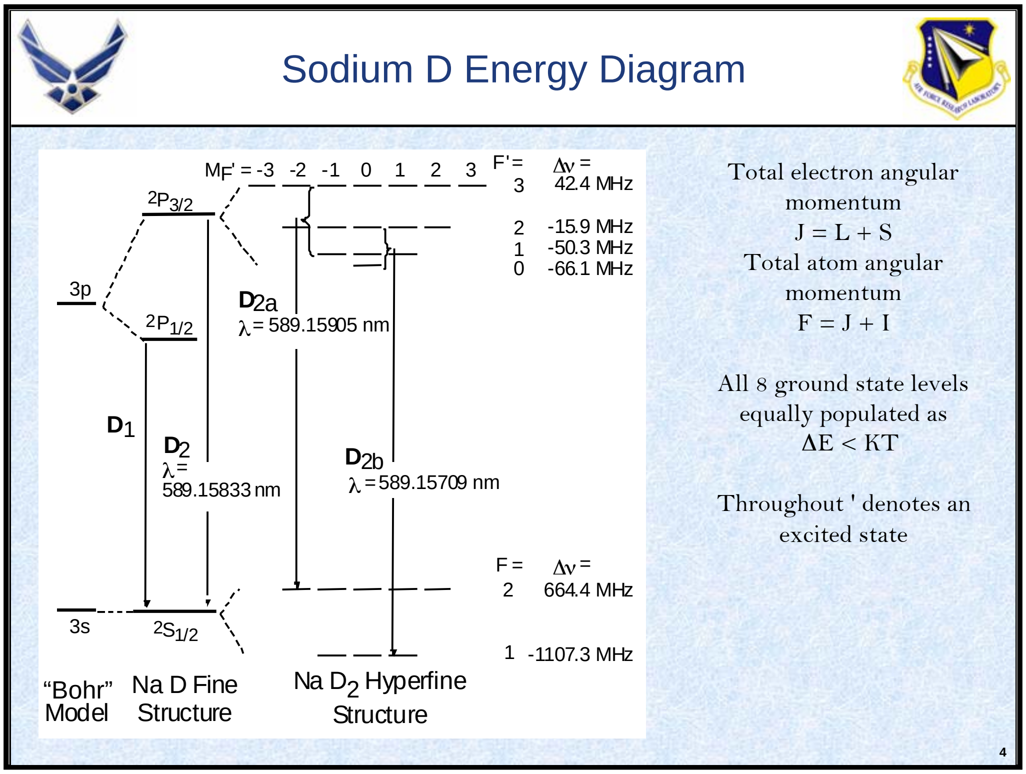
- Geometry
- Which transitions count
- Outline of calculations (without too much math)
- Results and comparison with return measurements
- Attempts to directly measure velocity profile due to recoil

We'll start out with a simple explanation of the geometry and coordinate axis. We'll show of the various sodium transitions which are relevant and which are not. Following that we will outline how the recoil profile was obtain, and compare the photon return from a guidestar with measurements. Finally we'll discuss our attempts to measure the velocity profile with a second Fasor.



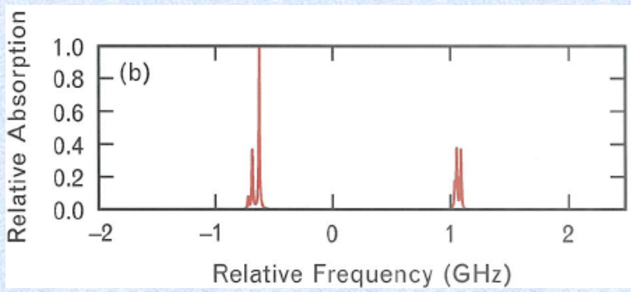
First we need to make a note of a small difference in symbols, v is velocity, ν is frequency.

When the narrow linewidth cw Fasor is tuned to the D2a line, only atoms with ~0.0 x velocity are excited. On absorption of a photon the atom gains the momentum of the photon. Since spontaneous emission on average is symmetrical, the atom gains no velocity from spontaneous emission. All stimulated emissions is along the +x axis and thereby cancels momentum gains from absorption. The spontaneous emission rate needs to be determined.



The sodium energy diagram showing increasingly detailed structure, starting with the Bohr model, and ending with the hyperfine structure. The fine structure is a result of the coupling between the orbital angular momentum **L** of the outer electron and its spin angular momentum **S**. The hyperfine structure is a result of the coupling of **J** with the nuclear angular momentum **I**. The atomic energy levels are shifted according to the value of *F*. Each of the hyperfine (*F*) energy levels contains 2*F* +1 magnetic sublevels, M_F, that determine the angular distribution of the electron wave function. The hyperfine structure of $^2P_{1/2}$ energy level is not shown.

Na Absorption Spectra



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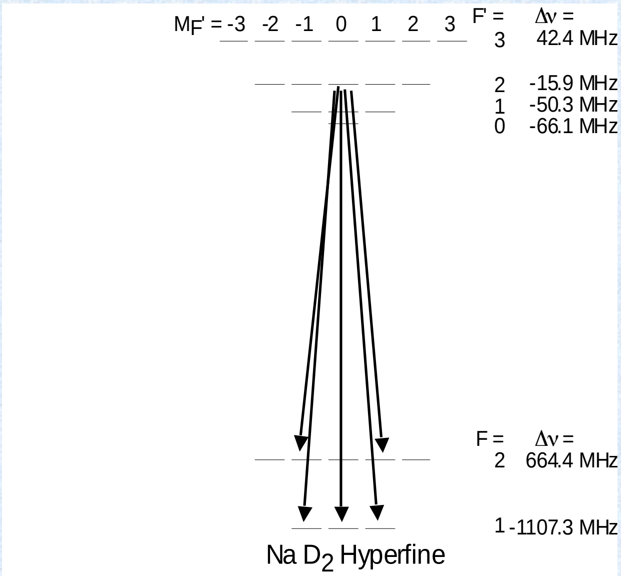
High resolution Sodium absorption spectra. The peaks on the left are due to D2a absorption, those on the right D2b. Each of the fine structure peaks is actually resolved into three hyperfine lines.

Transition Rules for Spontaneous Emission



<u>Selection Rules for allowed dipole transitions</u>

Spontaneous emission

$\Delta F = -1, 0, 1$

$\Delta M_F = -1, 0, 1$

$M_F=0$: $\Delta F=0$ & $\Delta M_F=0$ not allowed

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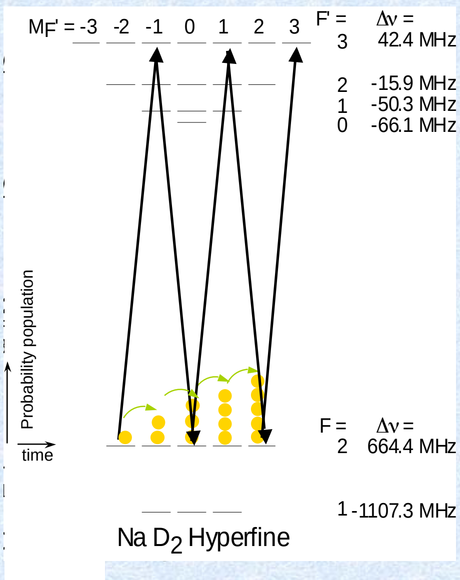
For spontaneous emission the transition rules are $\Delta F = -1, 0, 1$

$\Delta M_F = -1, 0, 1$. Note for M=0, $\Delta F = 0$, $\Delta M_F = 0$ simultaneously is not allowed.

Circular Polarization Excitation



<u>Selection Rules for allowed dipole transitions</u>

Optical absorption for circular polarization

$\Delta F = -1, 0, 1$

$\Delta M_F = +1$

After many absorption - decay cycles, atoms that start in the F=2 level end up in the F=2, $m_F = 2$ state. This state has the largest cross section

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For excitation using circularly polarized light, $\Delta F = -1, 0, 1$, but ΔM_F is limited to +1 for right handed circularly polarized light pumping, and to -1 for left handed circularly polarized light pumping. After a few absorption/emission cycles the atoms are ‘optically pumped’ to the $M_F = 2$ state. This allows sodium to be considered a two state atom.

Na Velocity Distribution

$$\frac{dN_{v_x}}{N dv_x} = \sqrt{\frac{m}{2\pi kT}} e^{-m v_x^2/2KT}$$

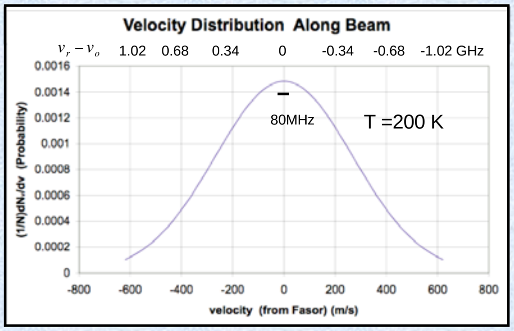
$$\langle v_x \rangle = 0$$

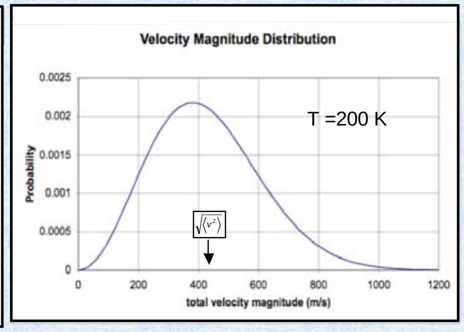
$$v_r = v_o (1 - v_x / c)$$

$$\frac{dN_v}{N dv} = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2KT}$$

$$\langle v^2 \rangle = \frac{3kT}{m}$$

Dwell time in stationary beam >1ms





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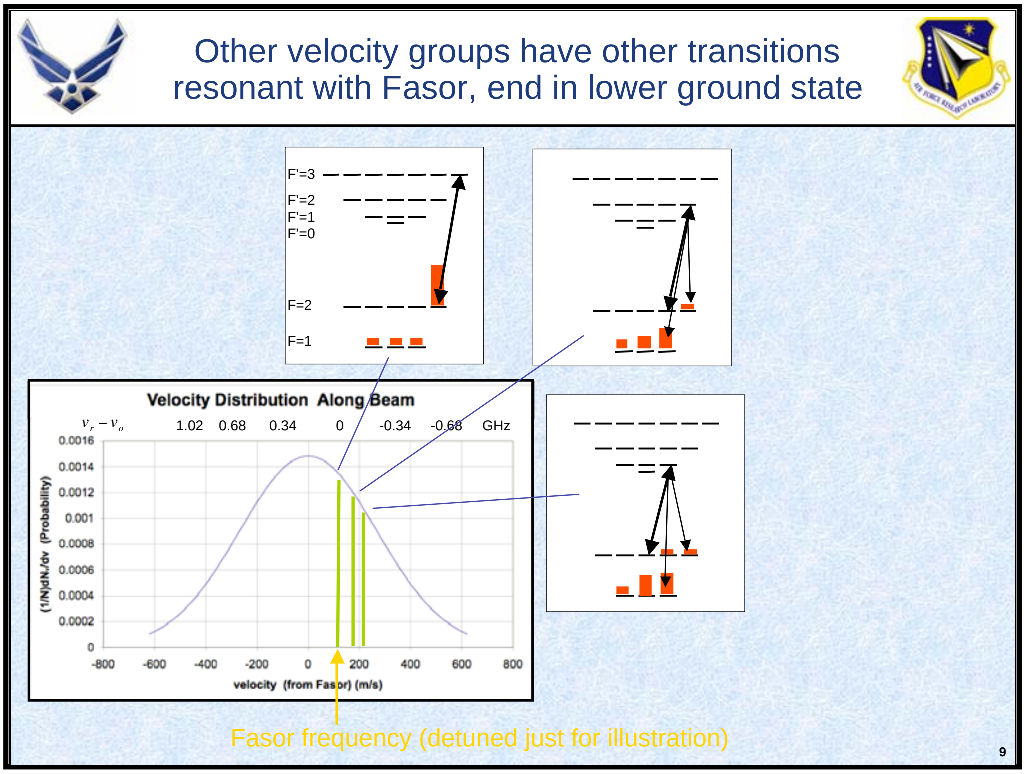
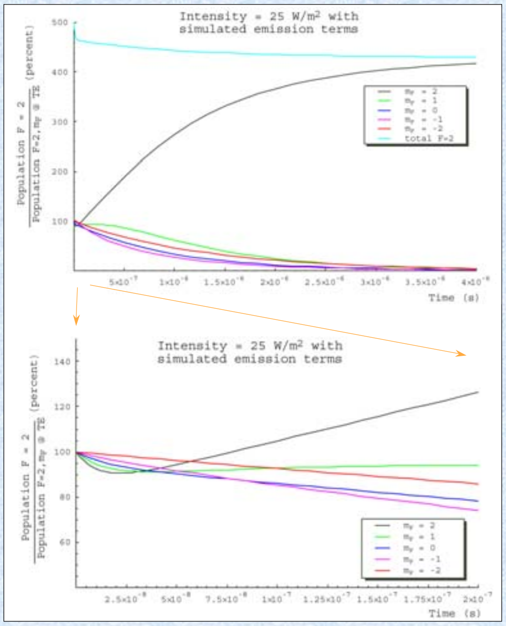
The natural linewidth of atomic sodium is only 10 MHz, however the mesospheric sodium can be excited by a sources as far as a gigahertz either side from the maximum. The cause of this is due to the distribution of velocities of individual sodium atoms, atoms with different velocities experience a different wavelength from the pump source due to the Doppler shift each experiences. The chart on the left shows the Maxwell Boltzman distribution of velocities away from the fasor source, the respective Doppler shift is shown on the top axis. Thus if the source is red shifted by 0.34GHz from line center, it would be the fraction of atoms with a velocity about 200 m/s toward the earth that would be resonant with the source. The number density at this velocity is only 12/15 ths of the maximum. Also shown is 80MHz wide width, we will later assume there is the same number of atoms throughout this width

For completeness the Maxwell Boltzman distribution of the total velocity *magnitude* is shown in the chart on the right. Although the average velocity along any given direction is zero, no atom has a velocity of zero or is standing still. For atoms with 0 x velocity, the mean velocity is close to 450m/s, implying a dwell time in a stationary 1 m wide beam of > 1ms, collsions at 58μs would change that.



For illustration we show the Fasor detuned from line center slightly. For the velocity class which F=2⇔F'=3 is resonant with the Fasor, all atoms are optically pumped into the m_F=2 sublevel. For the atoms where the F=2⇔ F'=2 transition is resonant, some atoms fall to the F=1 state and are no longer resonant, after a short time no atoms are left to be excited. The same goes for atoms in the velocity class where F=2⇔F'=1 is resonant. There are a few atoms left in F=2 , m_F =2 state that are not excited due to the fact there are no excited states available to which to be excited. More than likely these are transferred to excitable m_F states through spin relaxation transition and then soon end up in the F=1 ground state. Note that the separation between the states along the x axis is exaggerated.

How fast do atoms initially in F=2 transfer to F=2, m_F=2



At an intensity of less than $I_{sat}/2$, most the atoms that start in the F=2 level are swept to the F=2, m_F=2 sublevel in less than 4μs, much less than the collision time, ~50μs.

Close to Milonni’s calculations

Note this is for ‘on resonance’ excitation, off resonance and low intensities would take longer.

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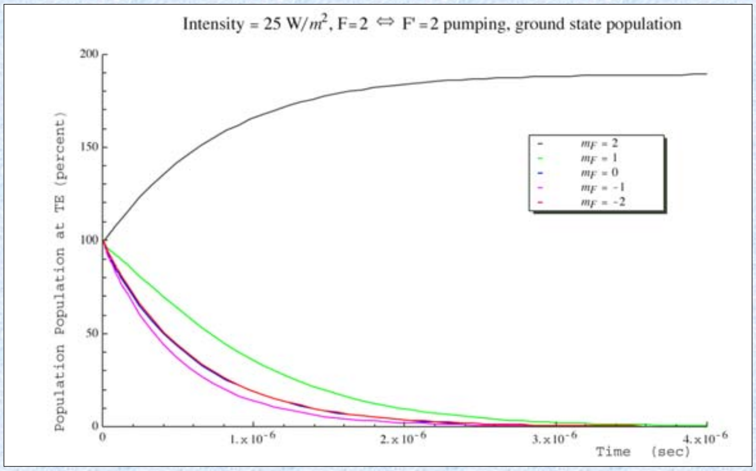
Optical pumping occurs within 4μs for intensity less than 1/2 Isat (25 W/m^2). The population of all five F=2, m_F states are shown as a function of time over 4μs. This is much faster then the collision time for sodium atoms in the mesosphere. The bottom plot is an enlargement of the first 0.2μs. These results are very close to Milonni’s, helping to verify our calculations. This is just for atoms ‘on line center’, the rate would be slower for atoms off line center and at lower intensities, like at the edge of the beam.

How fast do atoms in other velocity classes initially in F=2 transfer to F=1



At an intensity of less than $I_{sat}/2$, atoms in the velocity class where the Fasor is resonant with the F=2 ⇔ F'=2 transition, atoms that start in the F=2 level end up in the the F=1 level in less than 4μs, much less than the collision time, ~50μs. Note F=2, m_F=2 is not excited (forbidden transition)

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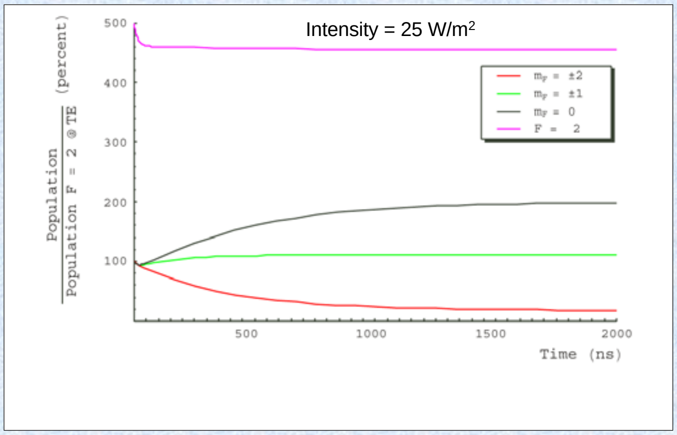
Atoms in other velocity classes where the F=2 <-> F'=2, and F=2<->F'=1 transitions are resonant decay eventually to the lowest ground state, F=1. Note that atoms can decay to F=2, m_F=2 sub state but are not excited out of it.

Steady State Distribution for Linear Polarization



- Steady State Reached in about 1/2 the time as for circular polarization
- More complicated as it is not a ‘two state’ atom
- Weighted averages were used to calculate σ_o and ε

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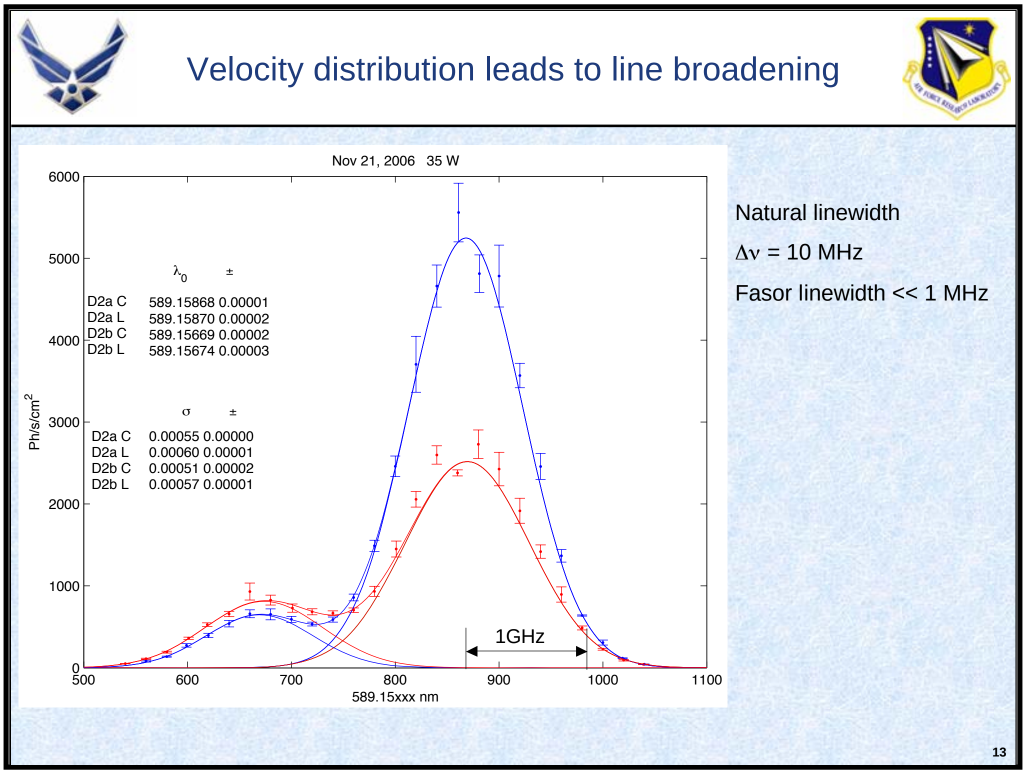
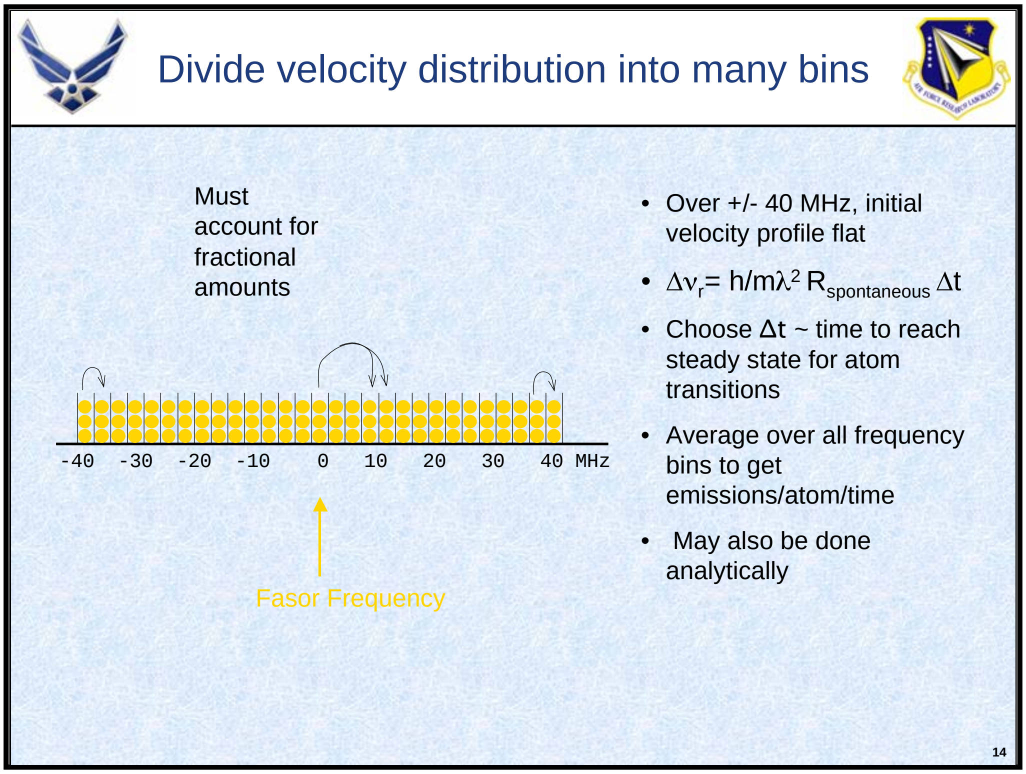
This is a plot of the F=2 mF states population probabilities against time when pumped by I = 25 W/m^2 linear polarization. Since at least three ground and three excited states are involved, weighted averages were used for σ_o and ε.



This plot is shown again to emphasis that for D2b, linear polarization gives a greater return than circular polarization. We feel that this should help us determine the transition rates between adjacent magnetic sub levels, m_F.



A brute force numerical process was used to track the velocities of sodium atoms under illumination. A velocity span of +/- 40 MHz from line center was chosen and divided into bins; at this small of span all bin start with the same population. An appropriate time step was also chosen. For each time step, the frequency shift was determined for each bin, and atoms shifted from bin to bin. The total number of emissions for each bin can added and divided by the total atoms to get the average emission/atom. Finally what needs to be determined is $R_{spontaneous}$ as a function of wavelength and intensity.

Steady State Spontaneous Emission Rate from Rate Equations for Two Level Atom

$$\frac{dN'}{dt} = -\frac{N'}{\tau} + \frac{I}{h\nu}\sigma(\nu_r - \nu_F)(N - N') = 0$$

Rate equation for excited state population

$$\sigma(\nu_r - \nu_F) = \frac{\sigma_0(\Delta\nu/2)^2}{(\nu_r - \nu_F)^2 + (\Delta\nu/2)^2}$$

Lorenzian line shape

$$R_{spon} = \frac{N'}{N_t\tau} = \left(\frac{1}{2\tau}\right)\frac{I/I_{sato}}{1 + 4(2\pi\tau(\nu_r - \nu_F))^2 + I/I_{sato}}$$

Spontaneous Rate/atom (same as Steck)

N' = Excited state

N = Ground state

N_t = total = N' + N

τ = life time = 16.25 ns

σ_0 = line center cross section = 1.66 x 10^{-9} cm^2

$I_{sato} = h\nu/2\sigma_0\tau$ = line center I_{sat} =65 W/m^2

ν_R = Doppler shifted resonance frequency

ν_F = Fasor frequency

$\Delta\nu$ = linewidth = $1/2\pi\tau$

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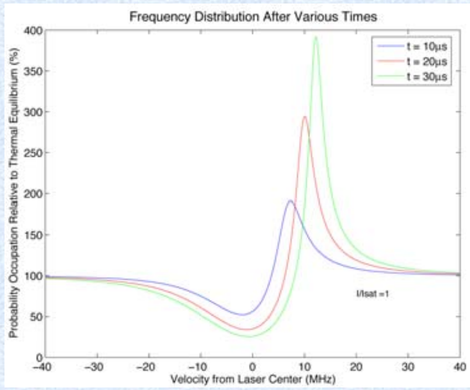
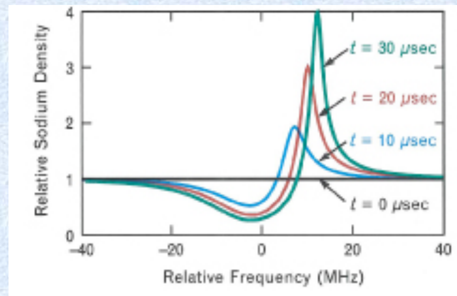
Assuming the atom is fully optically pumped, we use a simple rate equation for the two state atom, and assume the cross section has a Lorenzian line shape, the spontaneous emission rate can be determined as a function of intensity and detuning. Note again we have used ‘ to denote the excited state.

Compare results with T. Jeys





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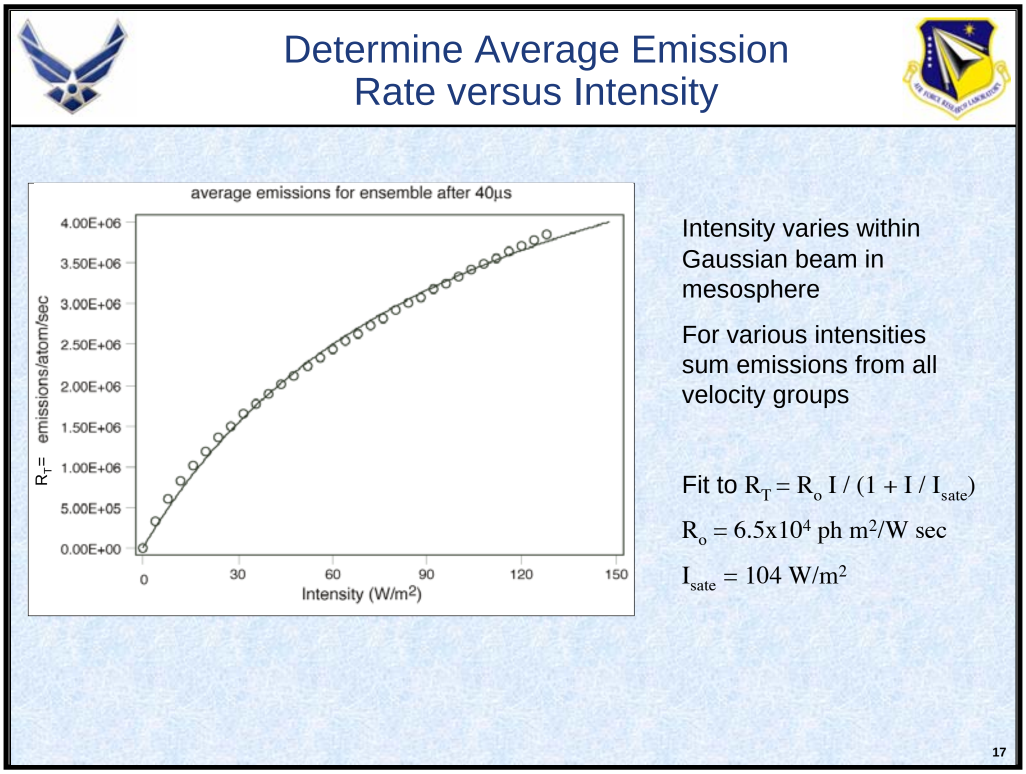
After just a few μs, the frequency (velocity) profile has a significant shape. It should be noted that collisions will tend to flatten the profile.



For a range of intensities this process was repeated, the average number of emissions/atom was determined for each intensity. After which R_{Ts} was fitted to a common saturation equation.

Calculated Photon Return

$$\frac{\text{Photons Scattered}}{\text{time}} = [Na] * \frac{5}{8} * 80\,MHz\,\frac{dN_x}{Ndv_x} * \int_0^{3\omega} \frac{R_0 I(r)/I_{sate}}{1 + I(r)/I_{sate}} 2\pi r dr$$

Column density

Fraction of Na atoms with electron in F=2

Fraction of Na atoms within Doppler Shift <40MHz~ .07

Integrate over guide star in mesosphere

Gaussian Intensity Profile, Spot size=2ω,
Atmosphere Transmission=T_a,
Fasor Power =P_F

$$I(r) = \frac{2\ P_F\ T_a}{\pi\ \omega^2} e^{-2r^2/\omega^2}$$

$$\frac{\text{Photons Detected}}{\text{area time}} = \frac{Photons\ \ Scattered}{time}\ 1.5\ \frac{T_a}{4\pi z^2}$$

T_a, z and ω corrected for telescope elevation

Directionality of emission from F'=3, m_F=3 level

Distance to guide star

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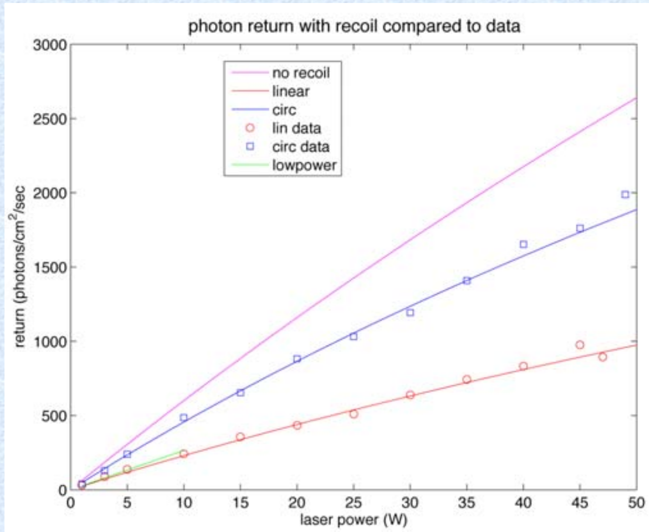
The photon return can be calculated by using the equation for R_T and integrating over the Gaussian beam in the mesosphere. This needs to be multiplied by the number of atoms intersecting the beam with Doppler shifts between +/-40 MHz of the fasor frequency. Note only 5/8ths of the atoms are in the F=2 upper ground state. Finally the return flux is determined by the area of a sphere with a radius the distance of the guidestar, the factor of 1.5 is a dipole directional emission factor for the F'=3, m_F=3 state on emission back toward earth.

Comparison with Measurements



Data taken May 8, 2006

Azimuth 180°, Elevation 70°

8.9° from B_e

Gardner: [Na] ≈

$3.8 \pm 1.7 \times 10^{13}/m^2$

Assumed [Na]=

$1.97 \times 10^{13}/m^2$

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The model is compared with data taken looking 8.9° to the geomagnetic field. The only adjustable parameter is the sodium column density. For early May at the SOR Gardner empirically estimates the [Na] at 3.8 x 10^{13} atoms/m^2 with an error of +/- 1.7. We have used a value of 1.97 x 10^{13} which is outside Gardner's deviation. More measurements need to be taken at different times of the year to ascertain if this was an unusually low [Na] night, although other data suggests it was not. Shown in magenta is the expected return if recoil is ignored. Shown in green is a low intensity curve where no optical pumping is assumed, all the F=2, m_F's remain having equal populations. Under these conditions circular and linear polarization give the same return because $\Sigma\sigma\varepsilon$ for the five involved states are the same. This model probably gives a higher return because it assumes no spin relation, i.e. no leakage from the optically pumped state back into neighboring m_F sub states.

Attempt to Measure Frequency Distribution due to Recoil

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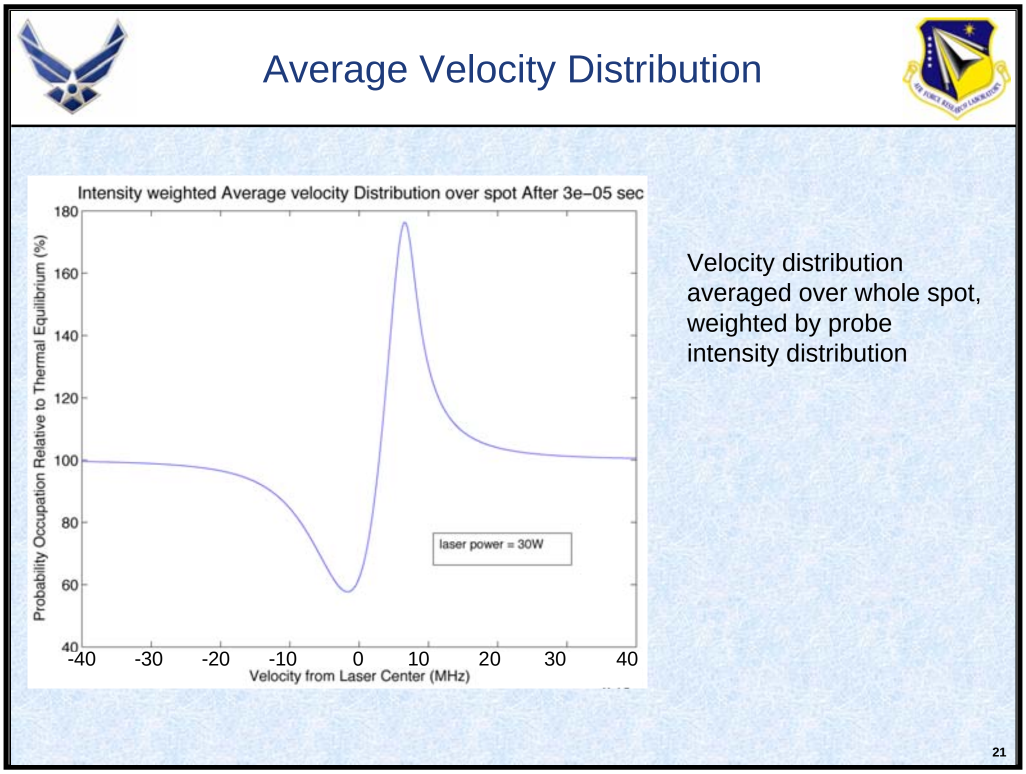
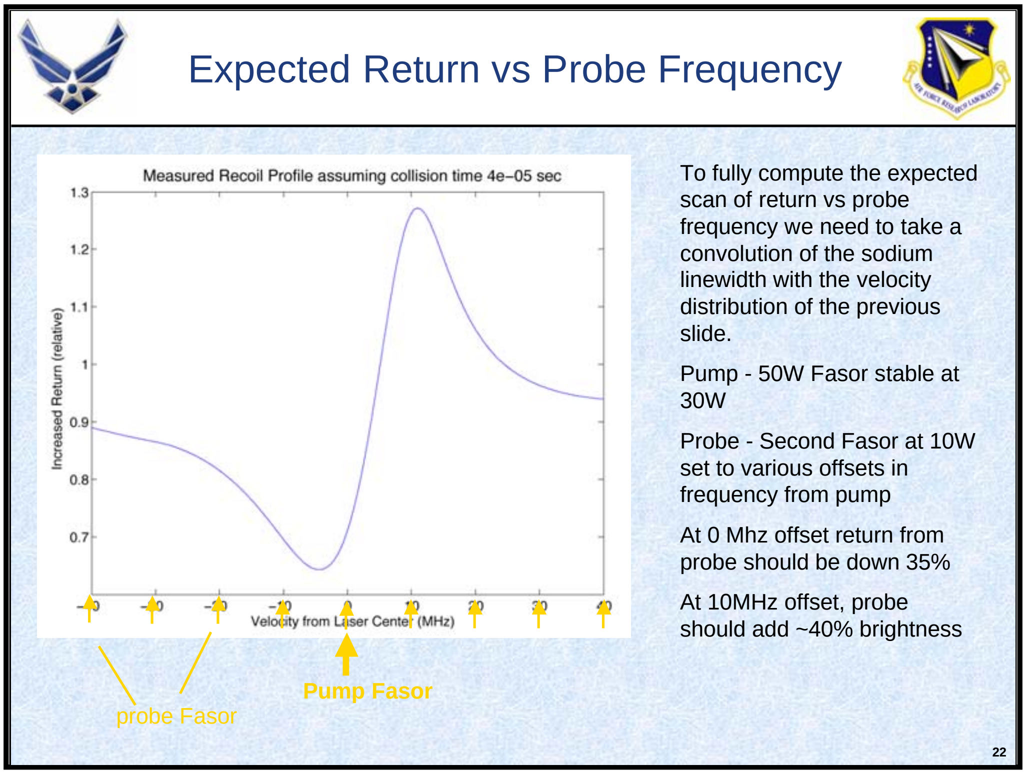
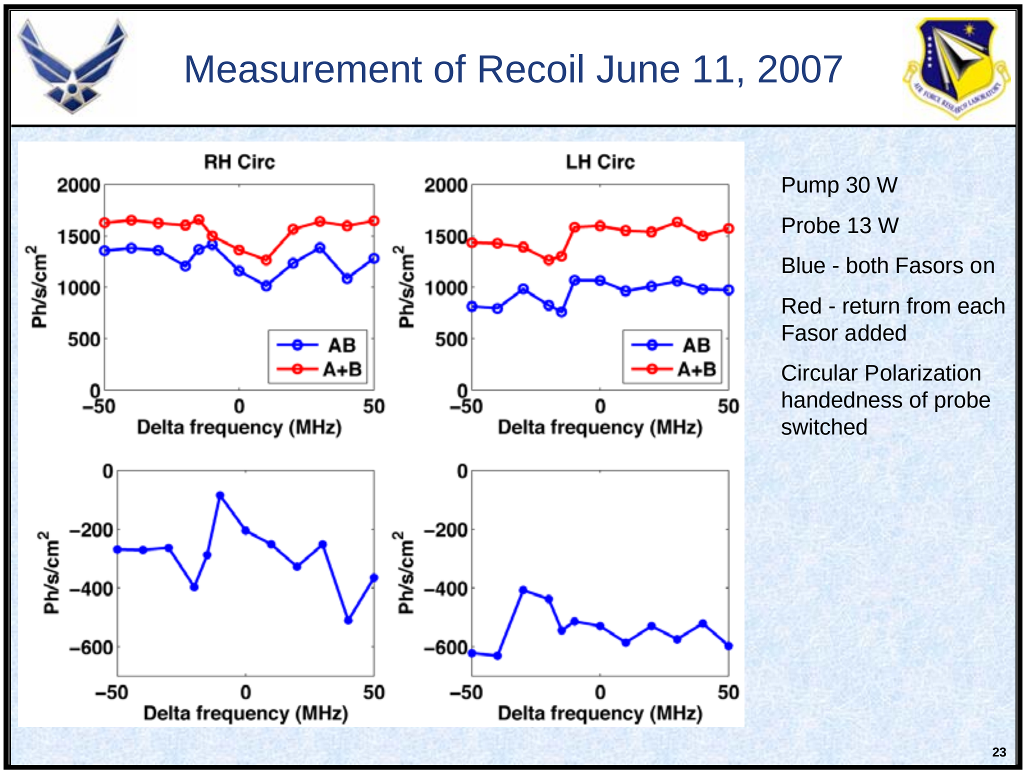
We made an attempt measure the recoil shifted frequency profile due to recoil by measuring the return using a second fasor while changing the the frequency of the second fasor.



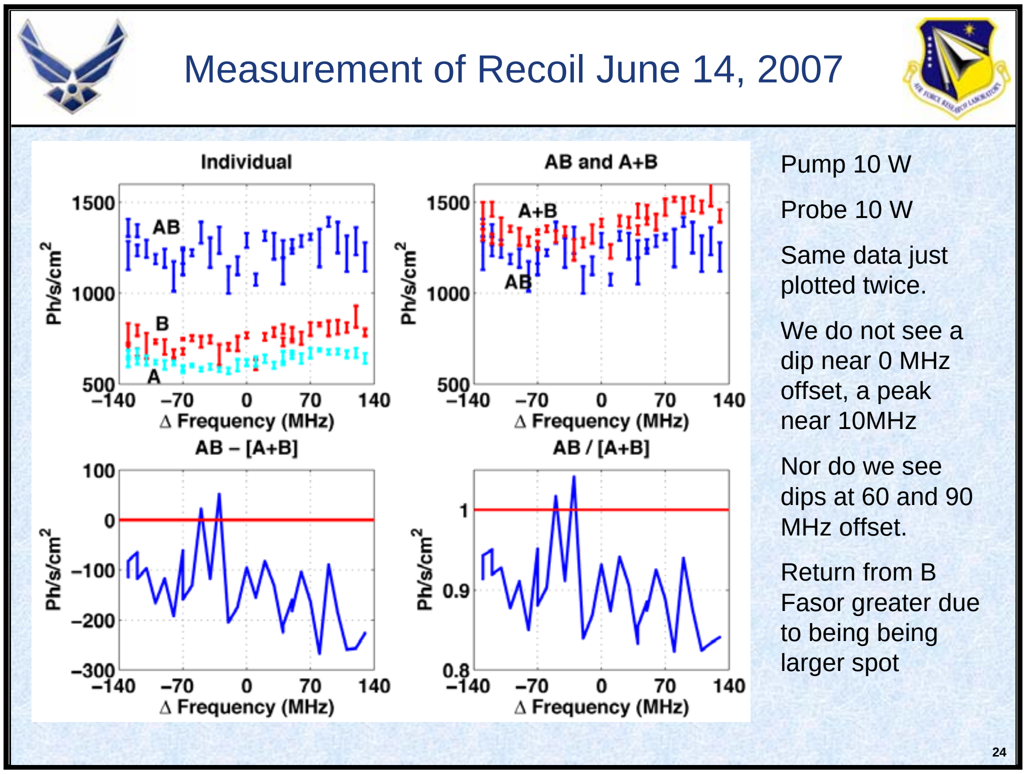
The velocity distribution was calculated at various values across the beam and then averaged using the probe beam’s intensity as a weighting factor.



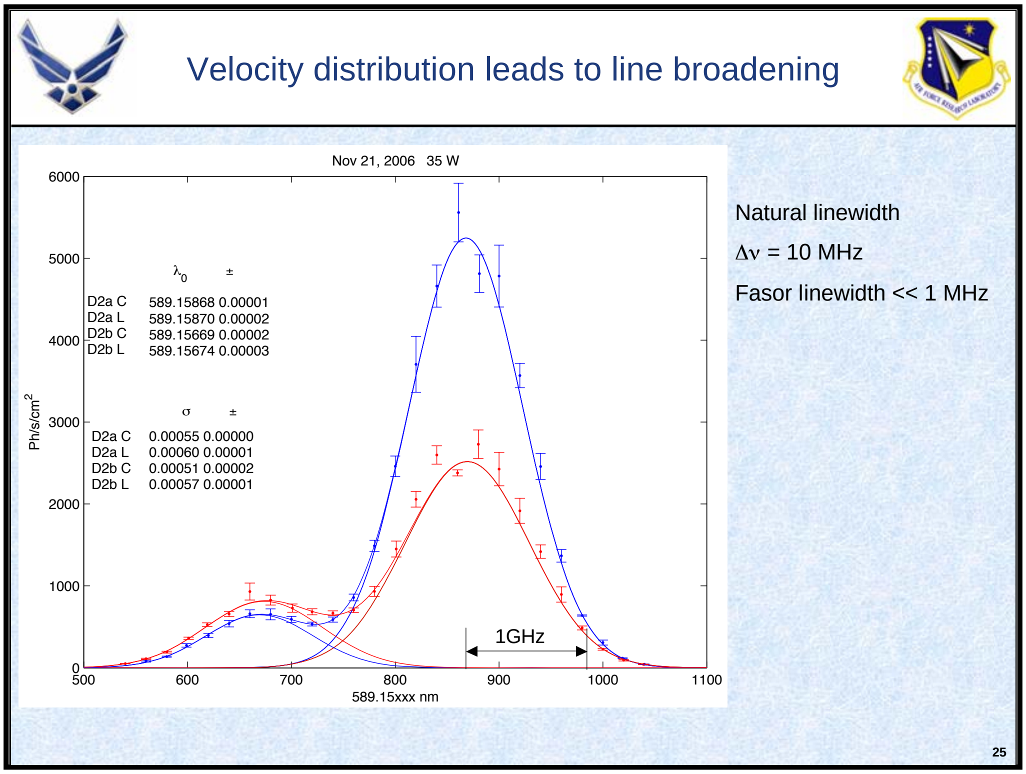
To fully compute the expected scan of return vs probe frequency we need to take a convolution of the sodium linewidth with the velocity distribution of the previous slide. The measurements were done at discreet frequency differences, not continuously.



On the first night we tried two difference senses of circularly polarized light from the probe beam, plotted are the returns from combined two Fasors and the sum of two taken individually. What was denoted as right handed polarization is obviously the correct setting.



On a second night both fasors were set to 10W. The second fasor gives a higher return as it has a larger diameter spot in the mesosphere and thereby less saturation. Note at ~58 and 92 MHz difference we did not see a large drop off in return either. Sources of errors in these measurements was maintaining the frequency difference between the two Fasors, which was done by hand. The beams from the two Fasors were sampled, combined on a high speed detector and the the beat note was determined by using a high speed FFT spectrum analyzer connected to the detectors output. The frequency difference would often change +/- 10 MHz during a measurement. We'd like to repeat these measurements while continuously varying the frequency difference using a PMT to measure the guide star brightness continuously. Lowering the probe beam power and chopping it while using a lock in to measure just the probe response would be a better measurement technique.



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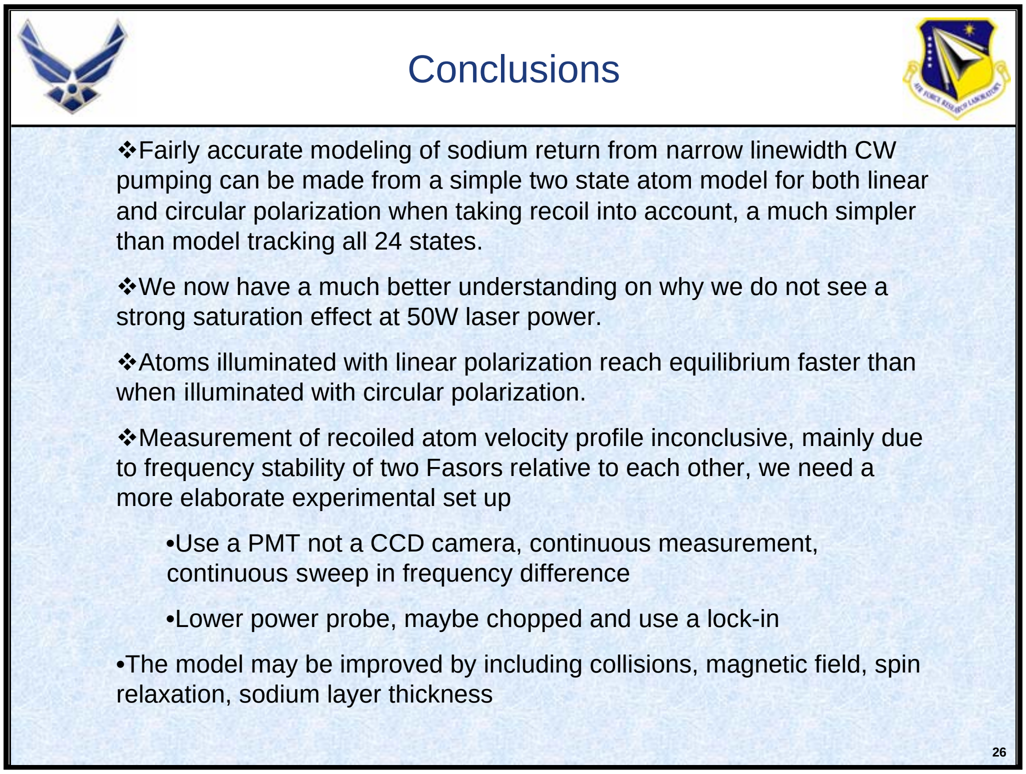

Basically summarize each point.

❖Fairly accurate modeling of sodium return from narrow linewidth CW pumping can be made from a simple two state atom model for both linear and circular polarization when taking recoil into account, a much simpler than model tracking all 24 states.

❖We now have a much better understanding on why we do not see a strong saturation effect at 50W fasor power.

❖Atoms illuminated with linear polarization reach equilibrium faster than when illuminated with circular polarization.

❖Measurement of recoiled atom velocity profile inconclusive, mainly due to frequency stability of two Fasors relative to each other, we need a more elaborate experimental set up

- •Use a PMT not a CCD camera, continuous measurement, continuous sweep in frequency difference
- •Lower power probe, maybe chopped and use a lock-in

•The model may be improved by including collisions, magnetic field, spin relaxation