Parametric Design of A MEMS Actuated Nanolaminate Deformable Mirror

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Parametric and Optimal Design: An organized way to design systems

• Come up with a conceptual architecture
• Determine goals of design
• Create a parametric model
  – Determine equations that describe important aspects of design in terms of parameters
• Use model to determine optimal design parameters
Parametric and Optimal Design Example: Define Problem

- **Project:** make a box to hold stuff
- **Goals:**
  - Volume of box must be $V$
  - Must use as little cardboard as possible
  - Must be rectangular
Parametric and Optimal Design Example: Determine Architecture

• Architecture: Rectangular prism
  – All six sides made of rectangular pieces of cardboard

• Meets requirements:
  – Holds stuff
  – Rectangular
Parametric and Optimal Design Example: Determine Goals

• Goals:
  – Volume of box is $V$
  – Surface area of box must be minimized
Parametric and Optimal Design Example: Create Parametric Model

\[ V = w \times h \times d \]
\[ A = 2\left( w \times h + w \times d + d \times h \right) \]

Which simplifies to:

\[ A = 2\left( w \times h + \frac{V}{h} + \frac{V}{w} \right) \]
Parametric and Optimal Design Example: Use Mathematical Model to Determine Solution

Solution: \( w = h = d = \frac{3}{\sqrt[3]{V}} \)
System Architecture

- Architecture
  - NL foil bonded to electrostatic actuator
Design Goals

• Applications will specify
  – Maximum voltage allowable
  – Minimum natural frequency
  – Minimum value of maximum displacement
  – Pixel size
  – Minimum cross-talk or minimum spatial frequency
Design Goals:
Cross-talk / Spatial Frequency

- NL foil stiffer than MEMS actuator
  - Behaves like a trampoline
  - Lots of cross talk
  - Not capable of high spatial frequency features

- MEMS actuator stiffer than NL foil
  - Behaves like a mattress
  - Low Cross Talk
  - Capable of High Spatial Frequency features
Design Goals:
Checker board pattern, a rational figure of merit

- Single point load
  - Difficult to model
  - Difficult to define figure of merit

- Checkerboard loading
  - Every other pixel actuated
  - Obvious figure of merit: Difference in displacement of adjacent pixels
Revised Design Goals:

- Applications will specify
  - Maximum voltage allowable
  - Minimum natural frequency
  - Minimum value of maximum displacement of actuated pixel
  - Maximum displacement of adjacent un-actuated pixel
  - Pixel size
  - Minimum cross-talk or minimum spatial frequency
Develop Parametric Model

• Parametric Model of Actuator
• Parametric Model of NL foil
• Combine into system model
Parametric Model: Determine Structure Type

- Regular plate
  - Resistance dominated by bending
  - Displacement less than thickness
- Membrane
  - Resistance dominated by stretching
  - Displacement much larger than thickness
- Plate with large deflections
  - Both bending and stretching are important
  - Displacement on order of thickness
Parametric Model of Actuator

Differential equation for large deflection plate

\[ \nabla^4 z = \frac{1}{D} \left( q(x) + \frac{1}{2} \frac{Eh}{(1 - \nu^2)} \left( \frac{\partial z}{\partial x} \right)^2 \frac{\partial^2 z}{\partial x^2} \right) + \left( \frac{\partial z}{\partial y} \right)^2 \frac{\partial^2 z}{\partial y^2} \]

Where

\[ D = \frac{Et^3}{(1 - \nu^2)} \]

\[ q(x) = \frac{1}{2} \frac{\varepsilon_0 V^2}{(g - z)^2} \, dx \, dy \]

BMC type actuator only bends in one dimension

\[ \nabla^4 z = \frac{1}{D} \left( q(x) + \frac{1}{2} \frac{Et}{(1 - \nu^2)} \left( \frac{\partial z}{\partial x} \right)^2 \frac{\partial^2 z}{\partial x^2} \right) \]

Solve with ODE solver

BMC actuator
Attachment point
Electrodes
Ends fixed
Results of Actuator Model

Position, Slope, Moment, Shear, and load for an actuator with $V=145$, $h=1.9053\mu m$ and $g=4.7\mu m$
Parametric Model of NL foil: Solution for Plates with Large Displacements

- 2-D case of actuator problem
  - Now a PDE
- Rayleigh-Ritz Solution
  - Solve for small displacement to determine shape
  - Solve for total energy to determine large displacement behavior
Parametric Model of NL foil: Navier Solution- Regular plates

- Equation for small displacements

\[ \nabla^4 z = \frac{1}{D} \left( q(x) + \frac{Eh}{(1 - \nu^2)} \left( \frac{\partial^2 z}{\partial x^2} \right)^2 + \left( \frac{\partial^2 z}{\partial y^2} \right)^2 \right) \]

- Assume load is sum of trigonometric functions

\[ q(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \left( \frac{m\pi x}{l_x} \right) \sin \left( \frac{n\pi y}{l_y} \right) \]

  - In this case a point load for each actuator

- Displacement will also be a sum of trigonometric functions

\[ z(x, y) = \frac{1}{\pi^4 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( \frac{a_{mn}}{\left( m^2 + n^2 \right)^2} \right) \sin \left( \frac{m\pi x}{l_x} \right) \sin \left( \frac{n\pi y}{l_y} \right) \]
Parametric Model of NL foil: Solve for Energy in Large Displacements

Solve for energy in plate

\[
dU_{\text{bending}} = \frac{1}{2} D \left\{ \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right\} - 2(1 - \nu) \left\{ \frac{\partial^2 z}{\partial x^2} \frac{\partial^2 z}{\partial y^2} - \left( \frac{\partial^2 z}{\partial x \partial y} \right) \right\} dx \, dy
\]

\[
dU_{\text{stretching}} = \frac{Et}{8(1 - \nu^2)} \left[ \left( \frac{\partial z}{\partial x} \right)^4 + \left( \frac{\partial z}{\partial y} \right)^4 + 2\nu \left( \frac{\partial z}{\partial x} \right)^2 \left( \frac{\partial z}{\partial y} \right)^2 \right] dx \, dy
\]

Use total energy to find restoring force
Parametric Model of System: Combine Actuator and NL models

- Describe DM as a system of springs
Natural Frequency of System

• Rayleigh method
  – Max potential energy equals max kinetic energy
• Solve for potential energy of both foil and actuator at max displacement
• Find kinetic energy of foil and actuator as a function of natural frequency
• Solve for natural frequency where maximum potential and kinetic energies are equal
Determine design: Finding a NL Foil to Work With an Existing Actuator

- Actuator exists
  - All actuator parameters already defined
- Must determine NL that allows sufficient deformation
Determine design:
Find an Optimal Actuator and Foil combination

• Determine constraints
  – Maximum and minimum dimensions
  – Maximum Voltage?
  – Max Displacement?
  – Minimum Natural frequency?

• Determine objective function
  – Voltage?
  – 1/Displacement?

• Use minimization software to find parameters that minimize objective function while preserving constraints