

SPECKLE STATISTICS AND PROPERTIES

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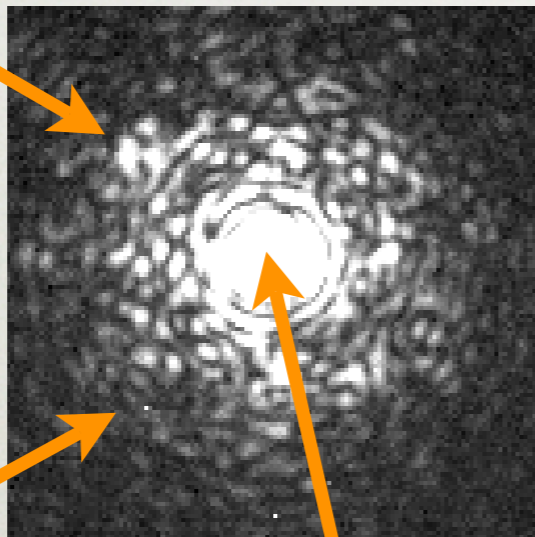
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A. SIVARAMAKRISHNAN, S. HINKLEY, B. OPPENHEIMER,

OUTLINE

STATISTICS AND PROPERTIES
IN DIRECT IMAGES

DIRECT

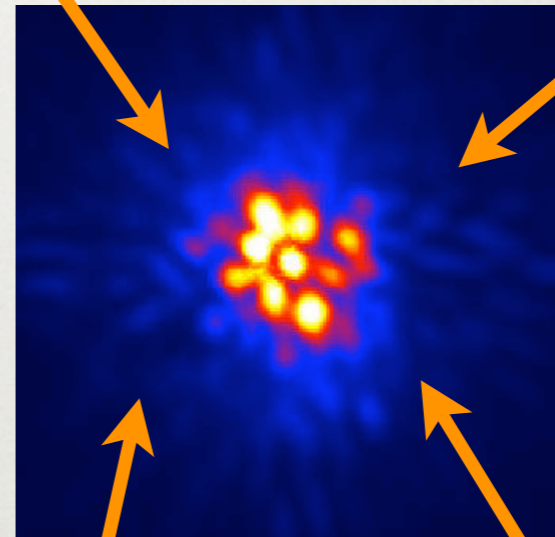


SIMULATION AND
AO DATA

STATISTICS AND PROPERTIES
AT THE CENTRAL POINT
(STREHL RATIO)

STATISTICS IN
CORONAGRAPHIC IMAGES

CORONAGRAPHIC



PROPERTIES OF STATIC
AND QUASI-STATIC
SPECKLES

EFFECT OF A CORONAGRAPH
ON SPECKLE NOISE

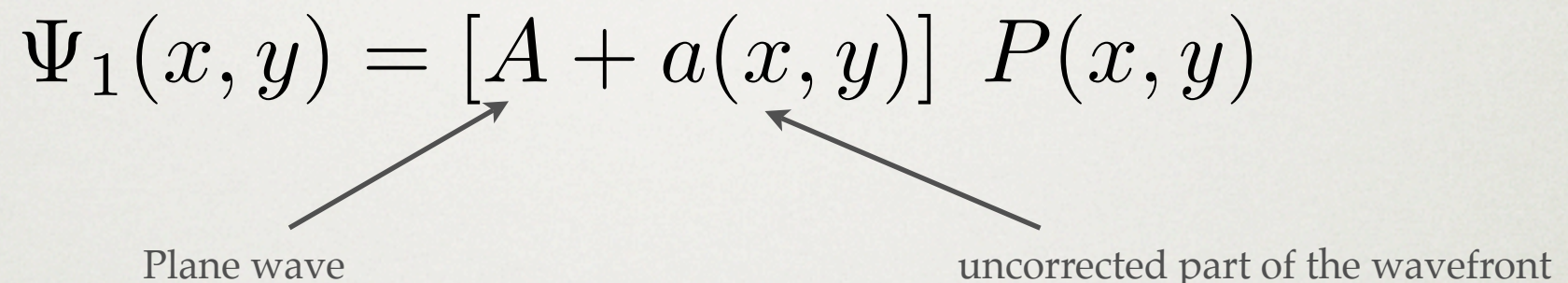
DETECTION LIMITS
(DYNAMIC RANGE)

STATISTICAL MODEL: PUPIL PLANE

- WAVE AMPLITUDE AT THE PUPIL PLANE:

$$\Psi_1(x, y) = [A + a(x, y)] P(x, y)$$

Plane wave

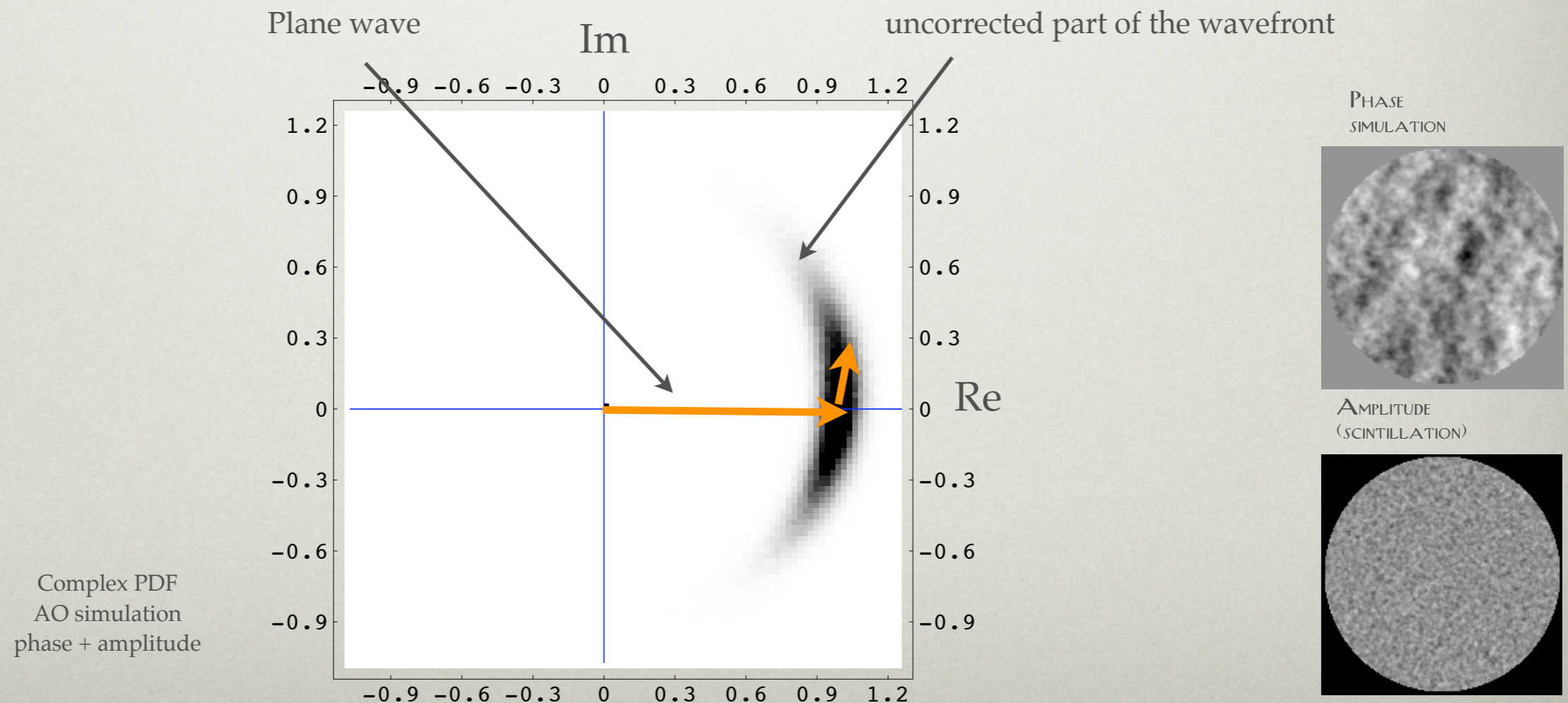


uncorrected part of the wavefront

STATISTICAL MODEL: PUPIL PLANE

- WAVE AMPLITUDE AT THE PUPIL PLANE:

$$\Psi_1(x, y) = [A + a(x, y)] P(x, y)$$



FOCAL PLANE COMPLEX AMPLITUDE

$$\Psi_1(x, y) = [A + a(x, y)] P(x, y)$$

$$\Psi_2(x, y) = \underbrace{A \times \mathcal{F}[P(x, y)]}_{\substack{\tilde{C}(\mathbf{r}) \\ \text{CONSTANT} \\ \text{DETERMINISTIC TERM}}} + \underbrace{\mathcal{F}[a(x, y) \times P(x, y)]}_{\substack{S(\mathbf{r}) \\ \text{SPECKLE} \\ \text{RANDOM TERM}}}$$

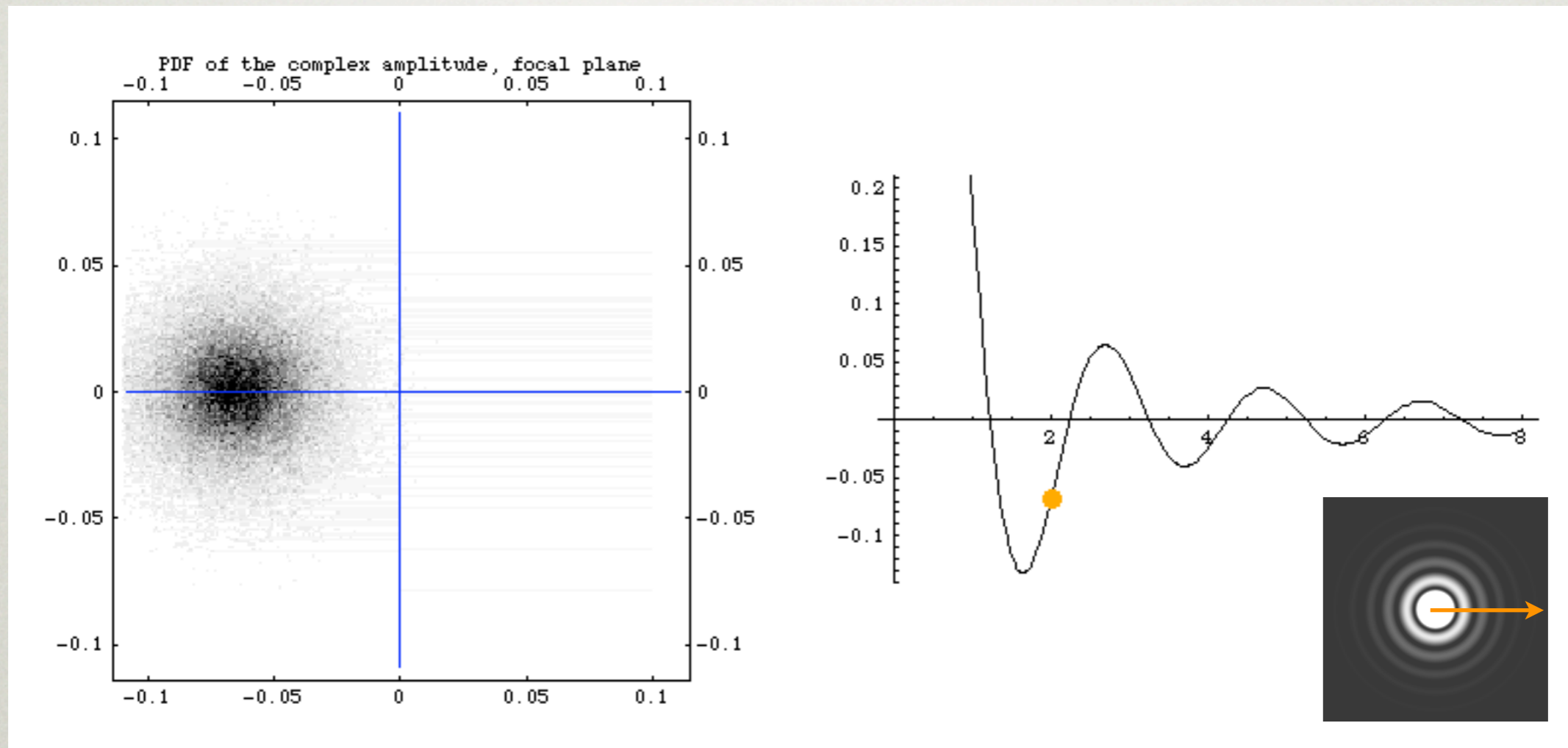
$$\mathcal{P}(\xi, \eta) = \frac{1}{\pi \langle |S(\mathbf{r})|^2 \rangle} \exp \left(\frac{-((\xi - \text{Re}[\tilde{C}(\mathbf{r})])^2 + (\eta - \text{Im}[\tilde{C}(\mathbf{r})])^2)}{\langle |S(\mathbf{r})|^2 \rangle} \right)$$

DECENTERED GAUSSIAN DISTRIBUTION

SAME SOLUTION AS FOR HOLOGRAPHIC SPECKLE (GOODMAN 1975)

FOCAL PLANE COMPLEX AMPLITUDE

$$\mathcal{P}(\xi, \eta) = \frac{1}{\pi \langle |S(\mathbf{r})|^2 \rangle} \exp \left(-\frac{(\xi - \text{Re}[\tilde{C}(\mathbf{r})])^2 + (\eta - \text{Im}[\tilde{C}(\mathbf{r})])^2}{\langle |S(\mathbf{r})|^2 \rangle} \right)$$



DECENTERED GAUSSIAN DISTRIBUTION

SAME SOLUTION AS FOR HOLOGRAPHIC SPECKLE (GOODMAN 1975)

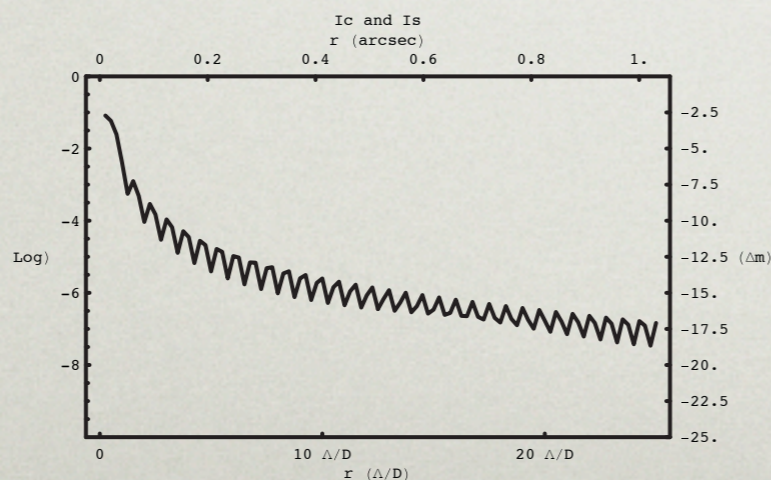
STATISTICAL MODEL: FOCAL PLANE

$$\mathcal{P}(\xi, \eta) \longrightarrow \mathcal{P}_I(I)$$

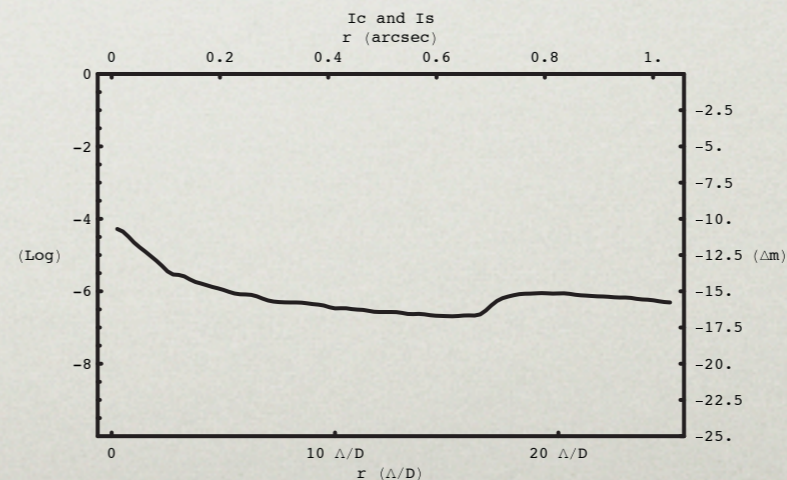
- RICIAN DISTRIBUTION

$$\mathcal{P}_I(I) = \frac{1}{I_s} \exp\left(-\frac{I + I_c}{I_s}\right) I_0\left(\frac{2\sqrt{I}\sqrt{I_c}}{I_s}\right)$$

$$\langle I \rangle = I_c + I_s \quad \sigma_I^2 = I_s^2 + 2I_s I_c$$

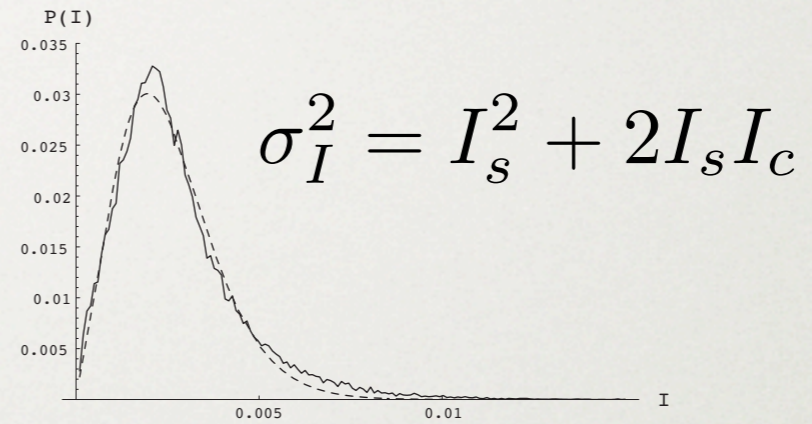
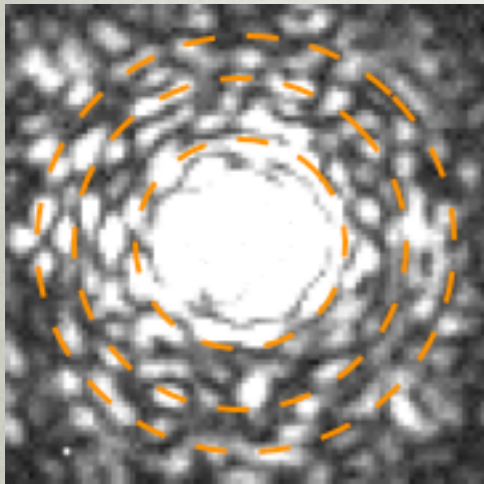


$$I_c = |\tilde{C}(\mathbf{r})|^2$$

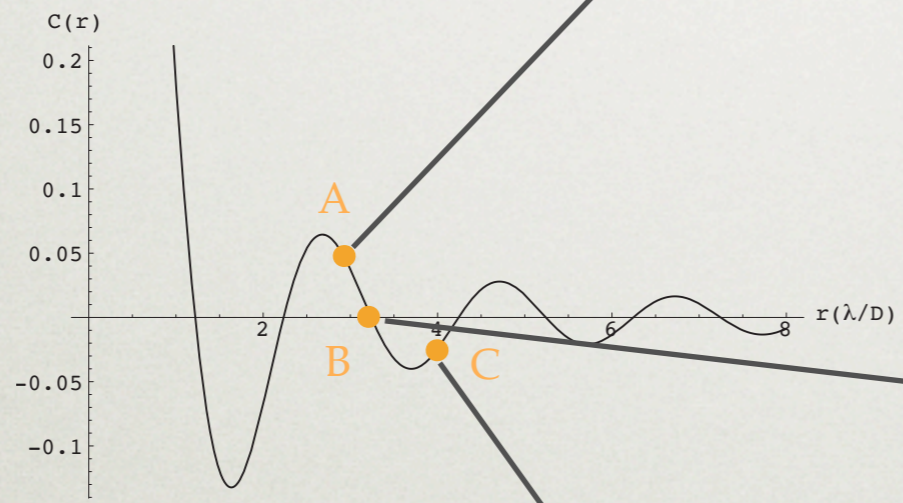


$$I_s = \langle |S(\mathbf{r})|^2 \rangle$$

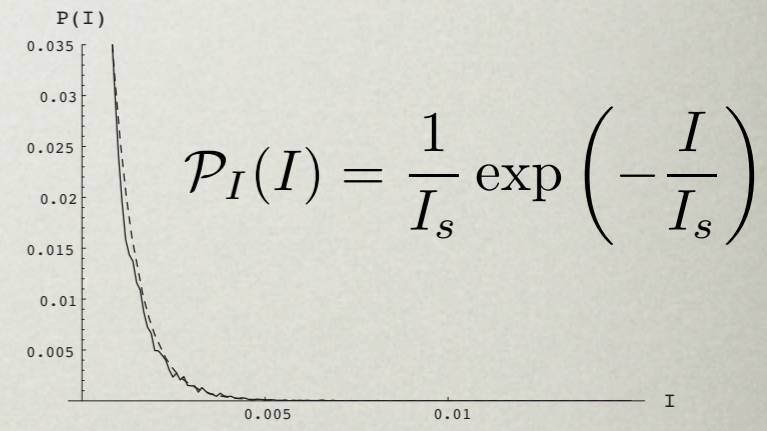
SPECKLE PINNING



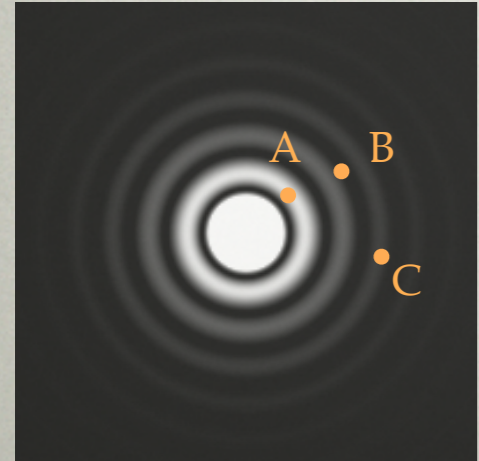
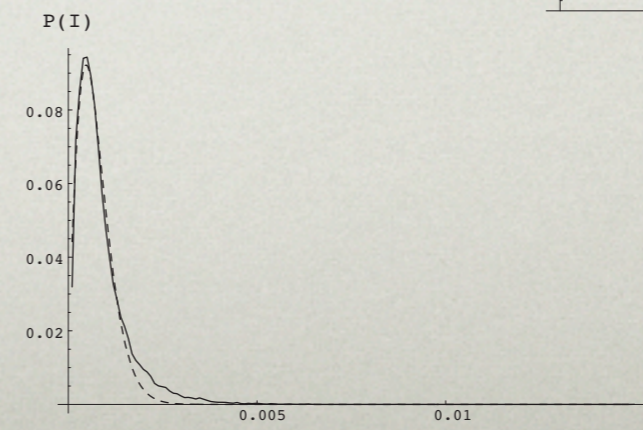
HIGH CONTINUOUS VALUE
AMPLIFICATION OF THE
FLUCTUATION



ZERO OF THE PSF
EXPONENTIAL STATISTICS

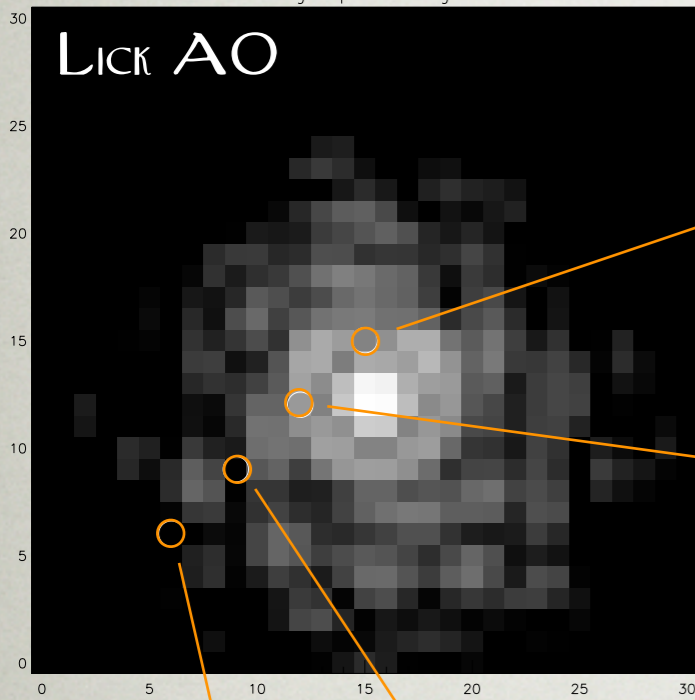


LOW VALUE OF THE
CONTINUOUS LEVEL

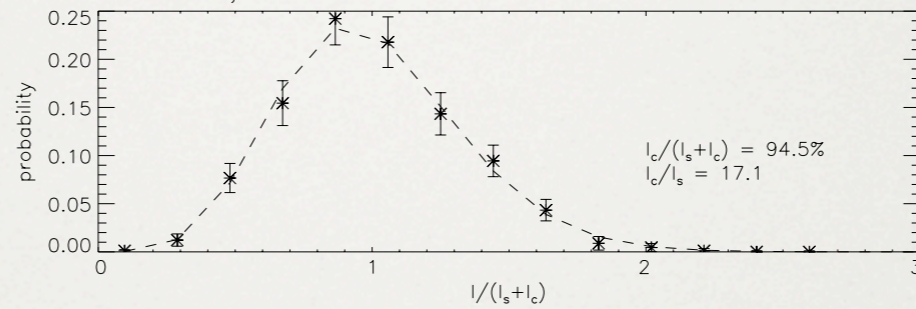


LICK AO DATA

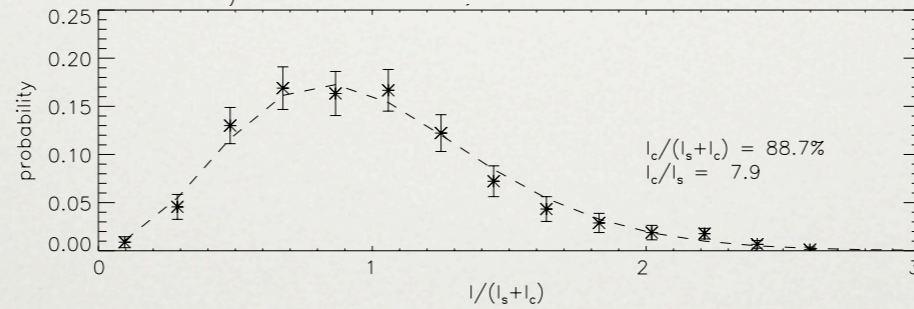
long-exposure image



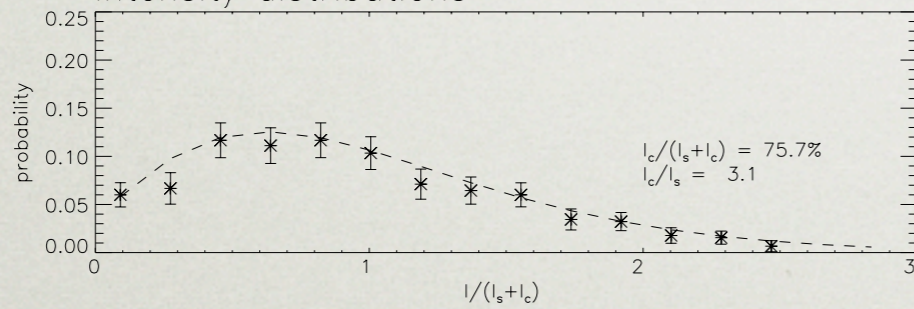
intensity distributions



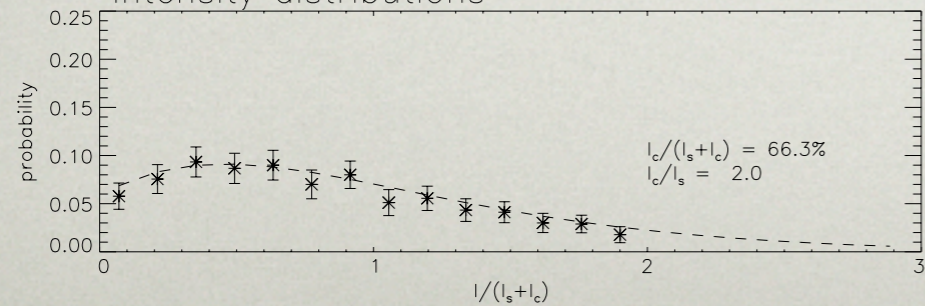
intensity distributions



intensity distributions



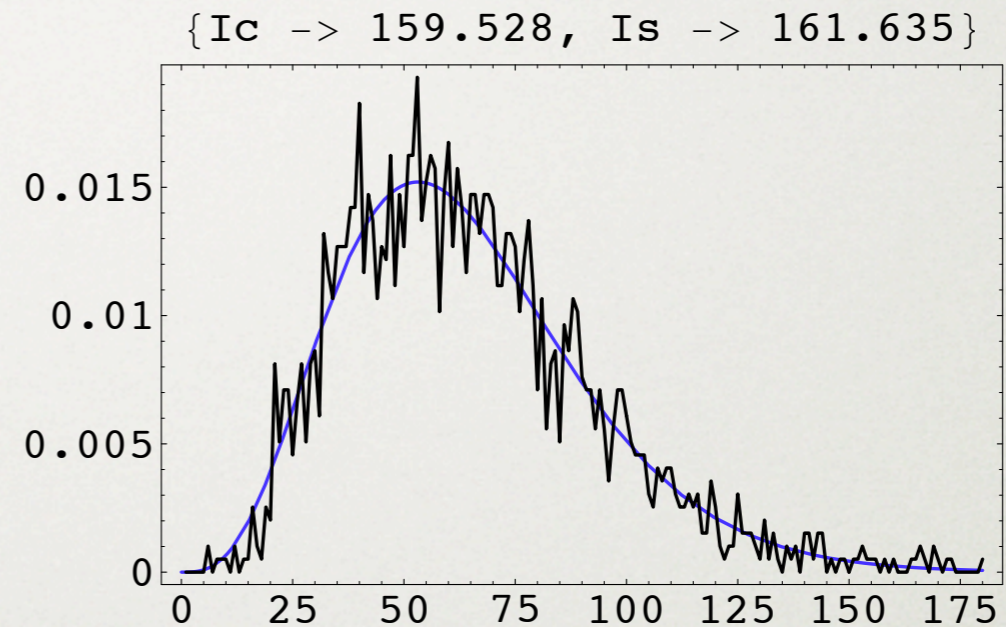
intensity distributions



FIRST VERIFICATION ON SKY OF
SPECKLES RICIAN STATISTICS:

FITZGERALD & GRAHAM (2005)

PALOMAR AO DATA



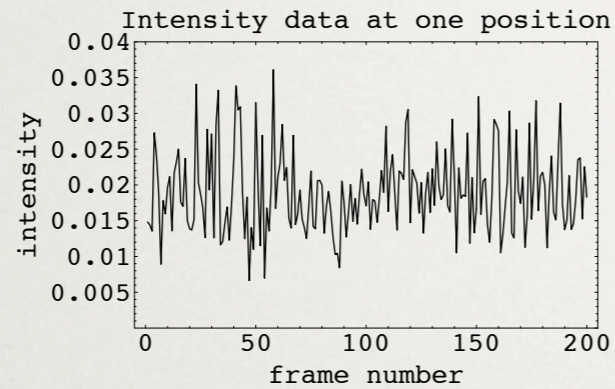
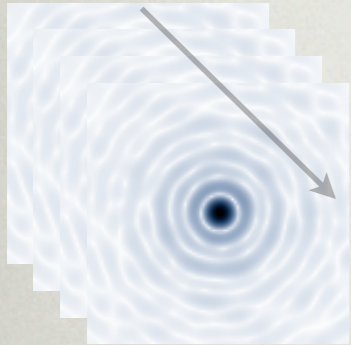
EXPOSURE TIME 120MS IN K BAND, NEED FOR A MODEL OF INTEGRATED SPECKLES

$$\mathcal{P}_I(I) = \frac{M}{i_s} \left(\frac{i}{i_c} \right)^{\frac{M-1}{2}} e^{-\frac{M(i+i_c)}{i_s}} I_{M-1} \left(\frac{2\sqrt{i}\sqrt{i_c}M}{i_s} \right)$$

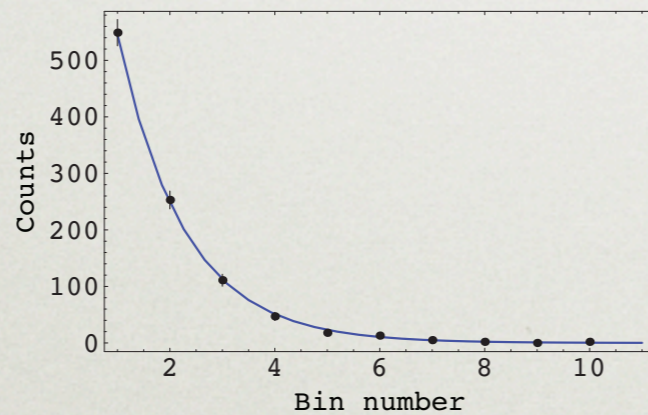
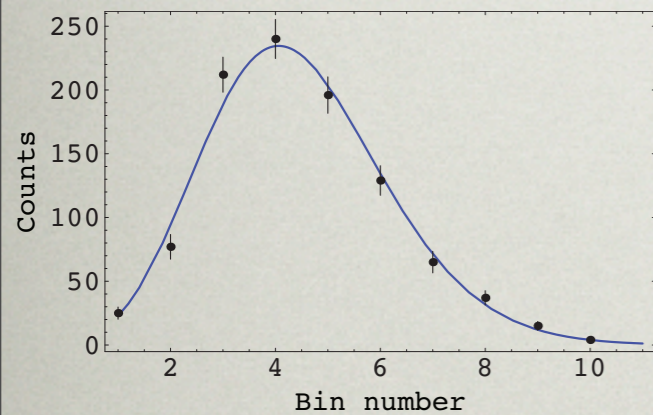
NON CENTRAL CHI SQUARE DISTRIBUTION OF ORDER M

RESULTS IN SIMULATIONS

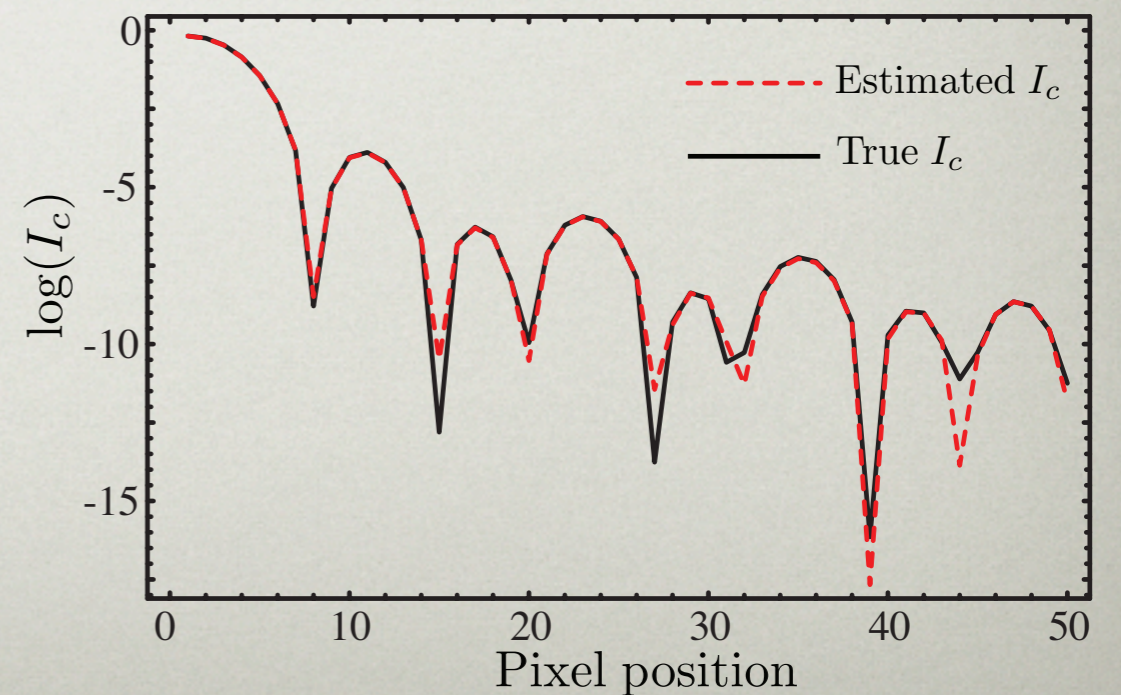
SIMULATED PSFs



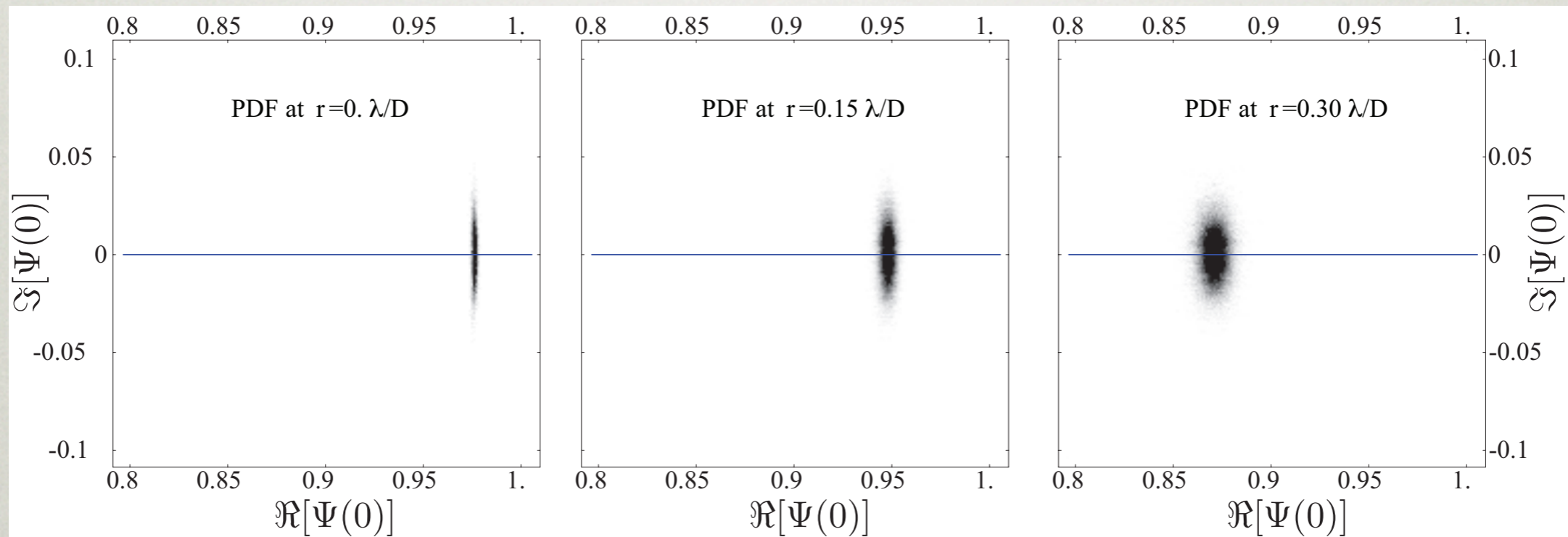
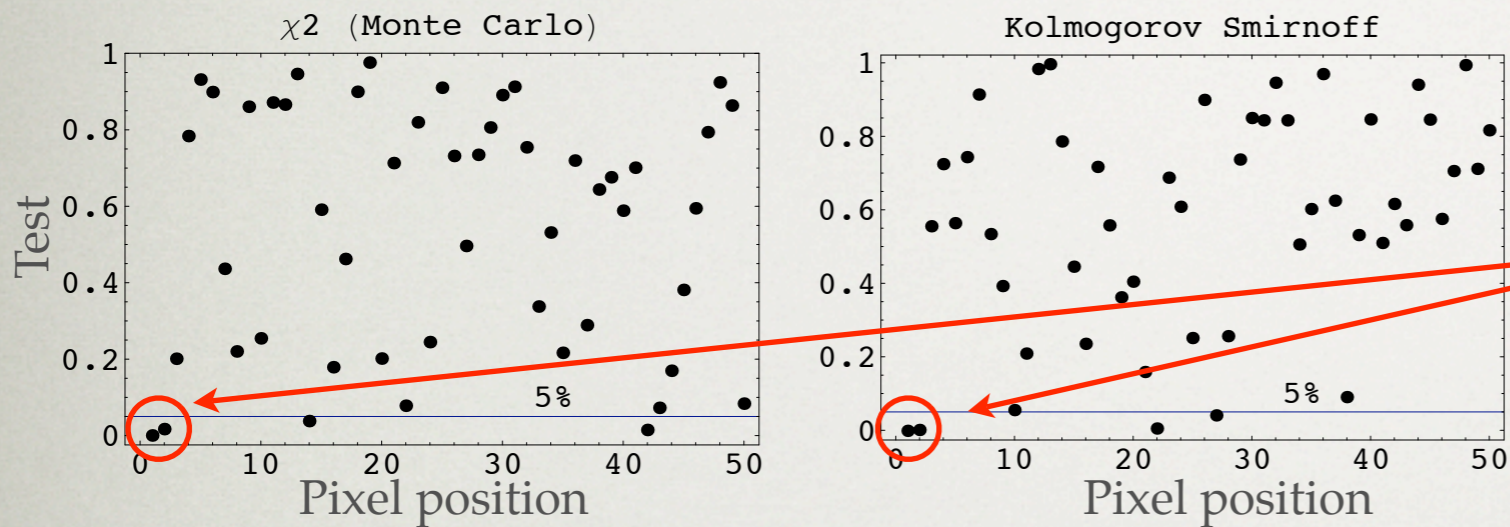
MAXIMUM LIKELIHOOD ESTIMATION
OF THE RICIAN PARAMETERS AND
TESTS STATISTICS



PERFECT PSF ESTIMATION FROM THE STATISTICS



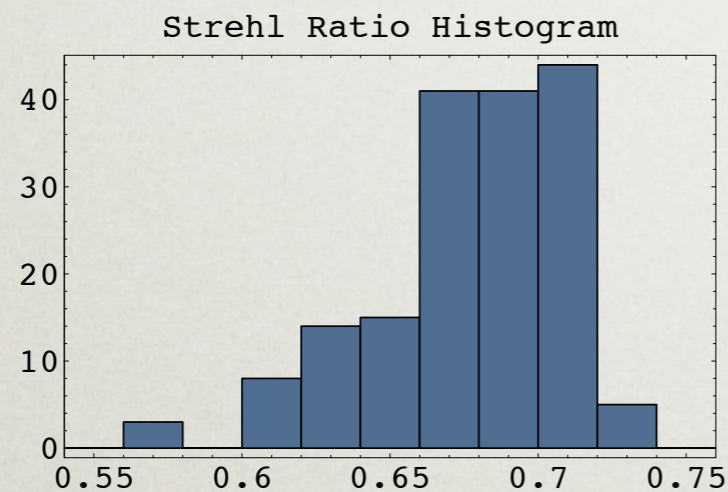
RESULTS IN SIMULATIONS



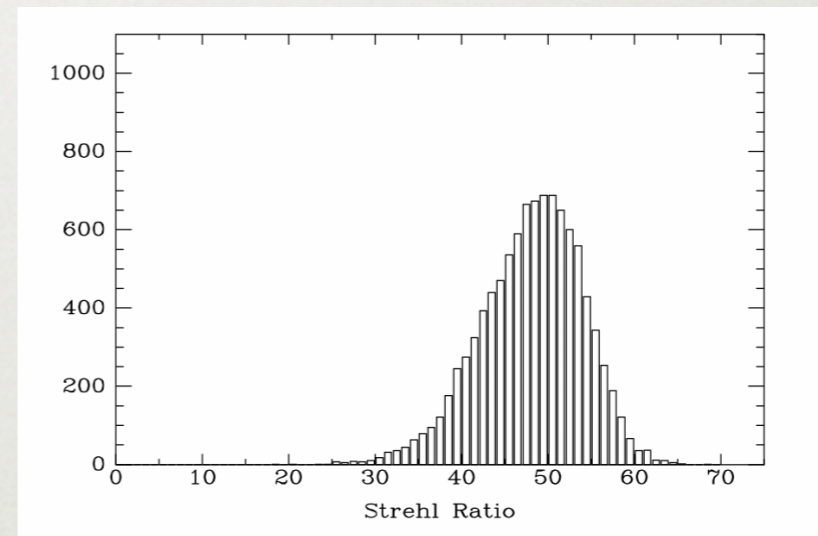
COMPLEX AMPLITUDE IS NOT CIRCULAR GAUSSIAN AT THE CENTER

STATISTICS AT THE CENTER

- AT THE CENTER OF THE PSF THE STATISTICS OF THE INTENSITY IS THE STATISTICS OF THE SR



PALOMAR AO (SOMMER & LLOYD)



LICK AO (GLADYSZ & CHRISTOU)

- NEW MODEL NEEDED TO DESCRIBE THE SR STATISTICS

STATISTICAL MODEL FOR THE SR

$$I = \left(1 - \int P(x) \frac{\Phi(x)^2}{2} dx\right)^2 + \left(\int P(x) \Phi(x) dx\right)^2$$

$$I \approx \left(1 - \int P(x) \frac{\Phi(x)^2}{2} dx\right)^2$$

$$\Psi(0) \approx \left(1 - \int P(x) \frac{\Phi(x)^2}{2} dx\right) e^{J\varphi}$$

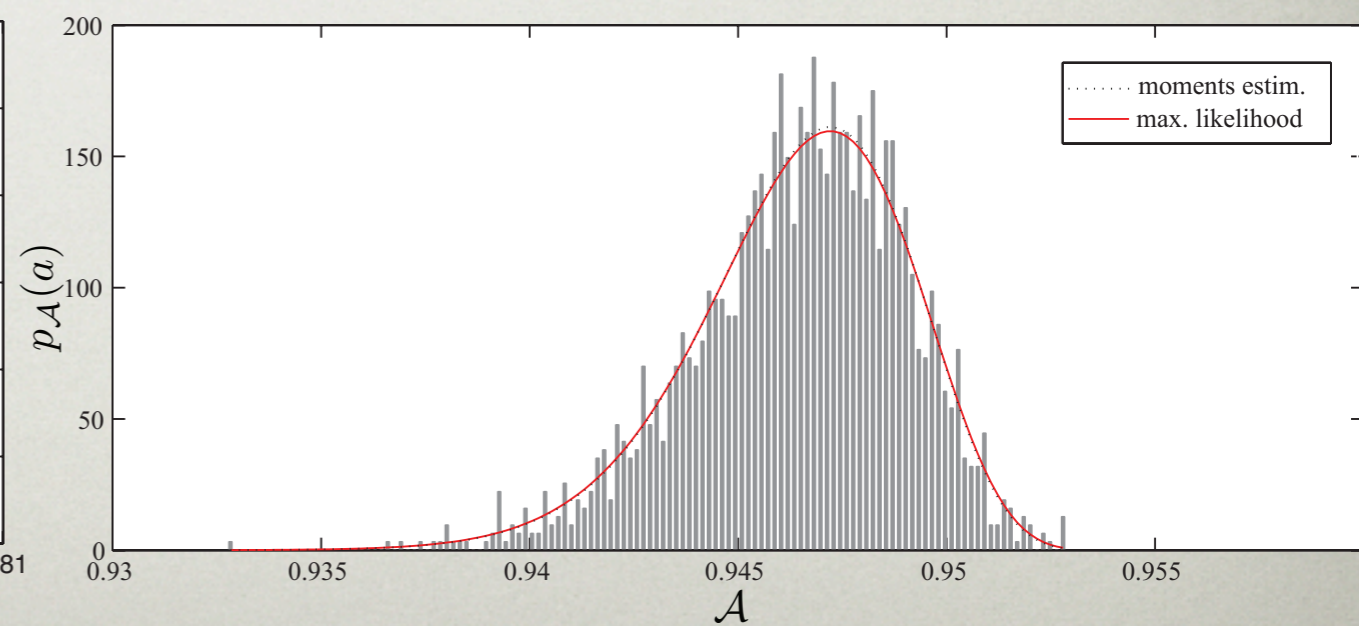
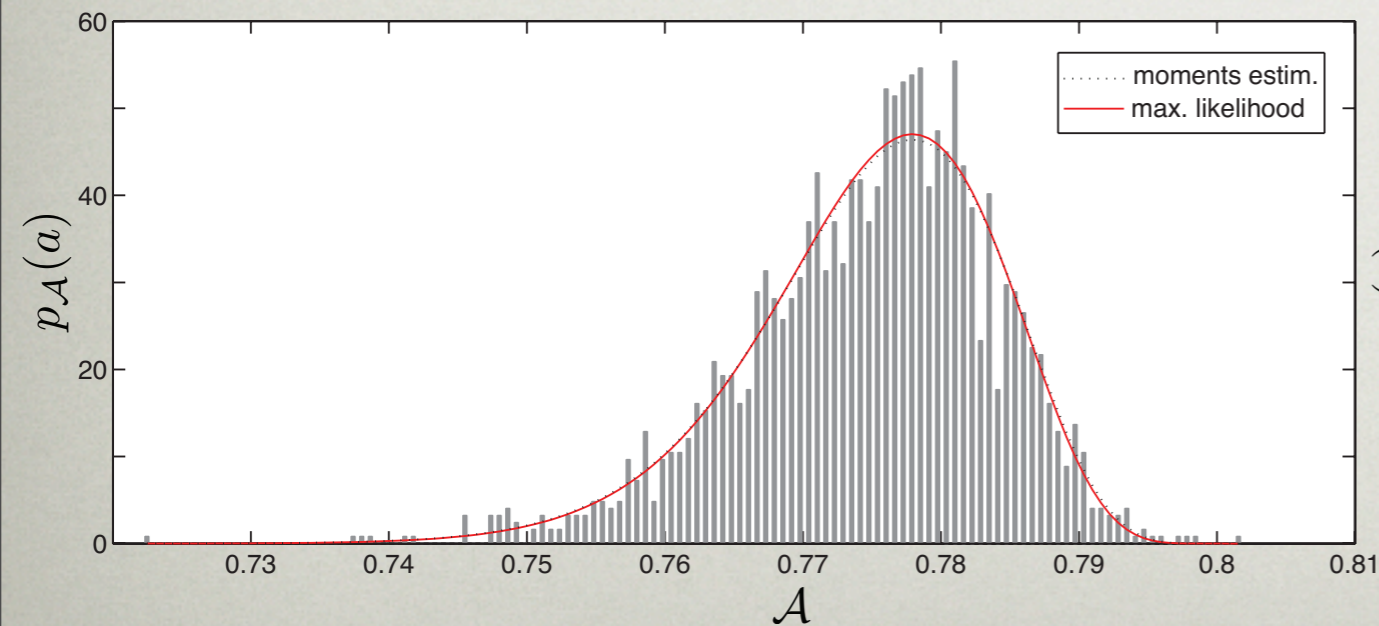
$$\Psi(0) = -\mathcal{A} e^{J\varphi} \quad \mathcal{A} \sim \Gamma(k/2, \sigma^2, -1)$$

THREE PARAMETER NON CENTRAL GAMMA DISTRIBUTION

MODEL OF THE COMPLEX AMPLITUDE

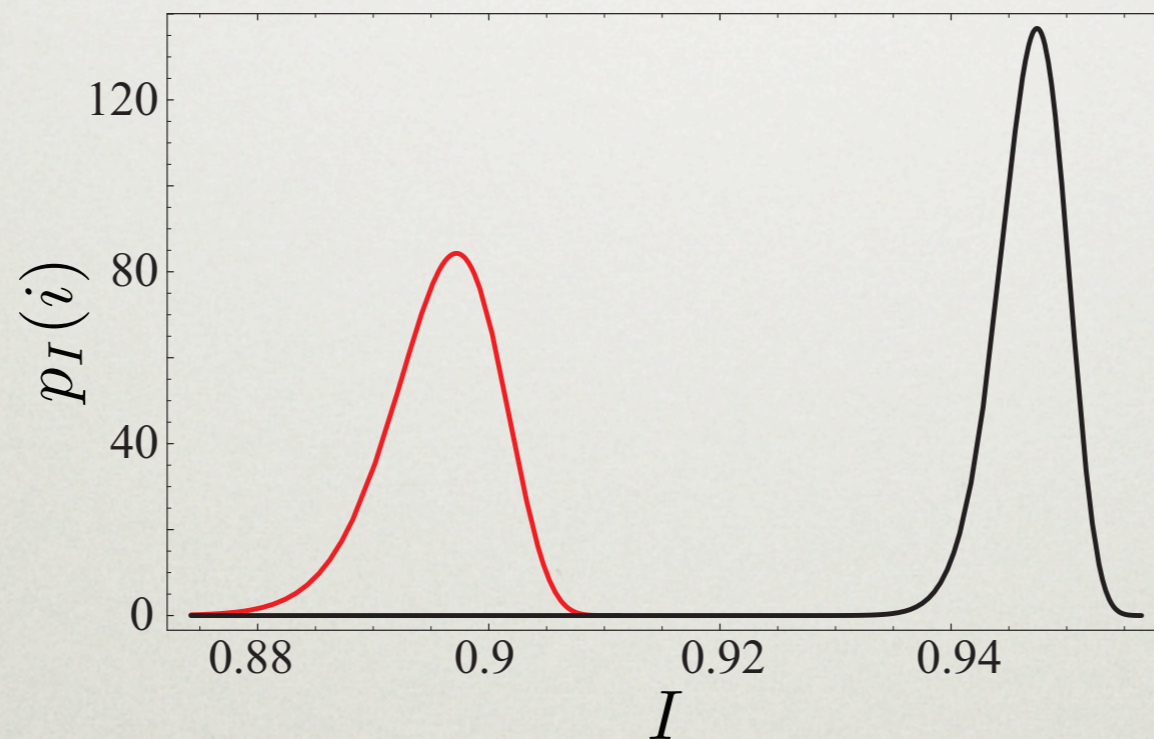
THE OPPOSITE OF THE COMPLEX AMPLITUDE AT THE CENTRAL POINT FOLLOWS
A THREE PARAMETER NON CENTRAL GAMMA DISTRIBUTION

$$p_{\mathcal{A}}(a) = \frac{(a - \gamma)^{\alpha-1} \exp(-(a - \gamma)/\beta)}{\beta^{\alpha} \Gamma(\alpha)}, \quad \alpha > 0, \beta > 0, a > \gamma.$$



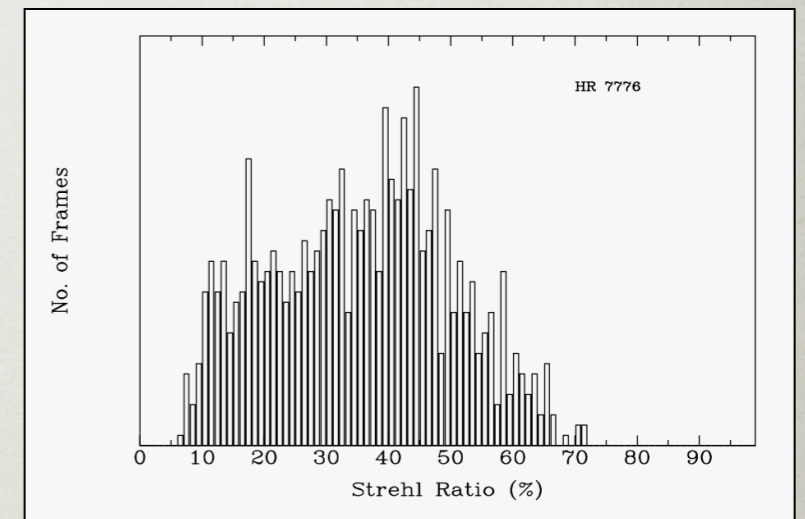
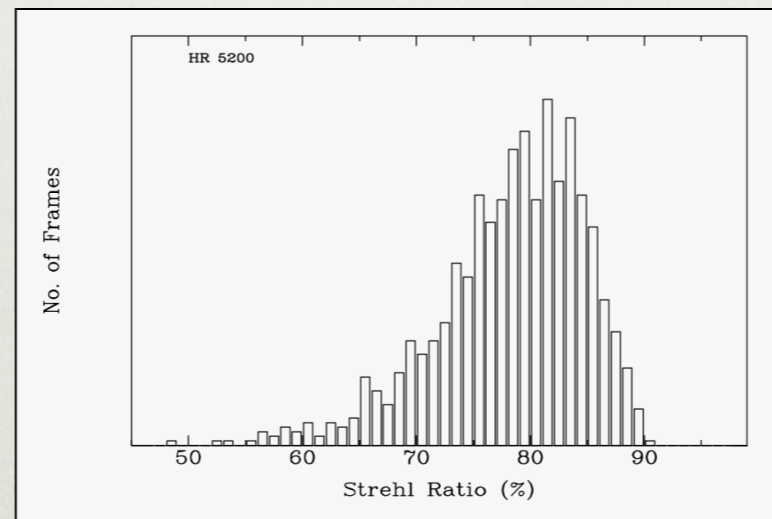
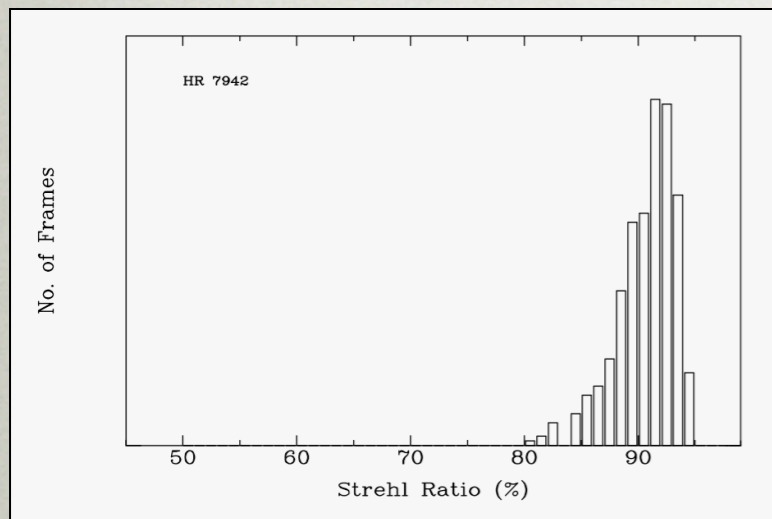
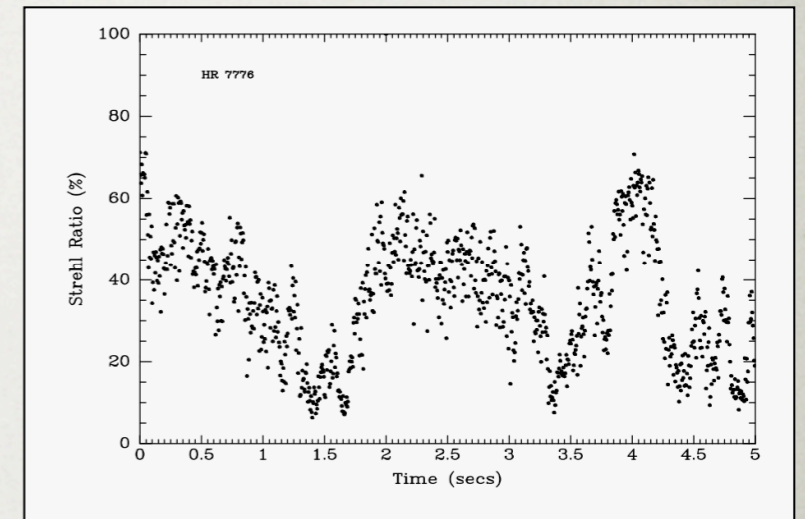
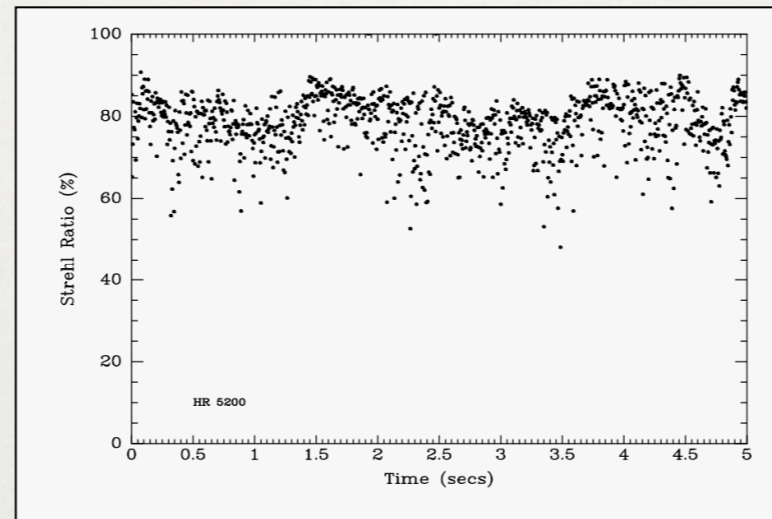
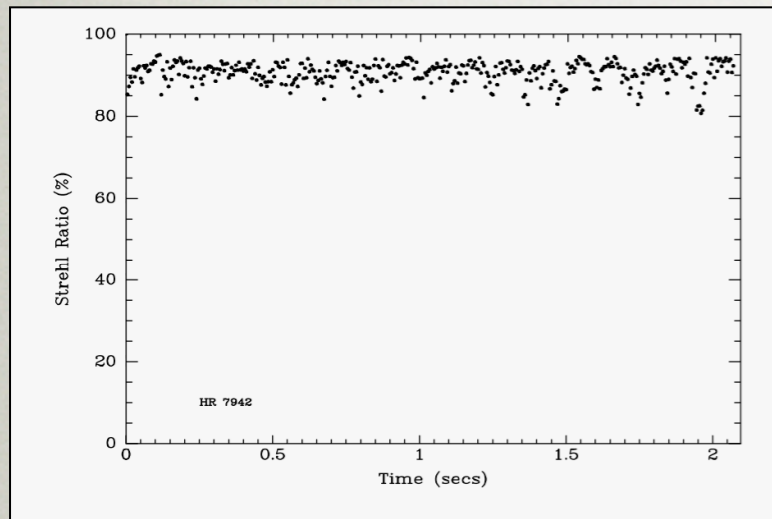
SR DISTRIBUTION AND ESTIMATOR

$$p_I(i) = \frac{1}{2\sqrt{i}} \left(p_{\mathcal{A}}(\sqrt{i}) + p_{\mathcal{A}}(-\sqrt{i}) \right), \quad i > 0$$



$$SR = \mathbb{E}[\mathcal{A}^2] = \alpha\beta^2 + (\alpha\beta + \gamma)^2$$

AEOS DATA FROM WFS

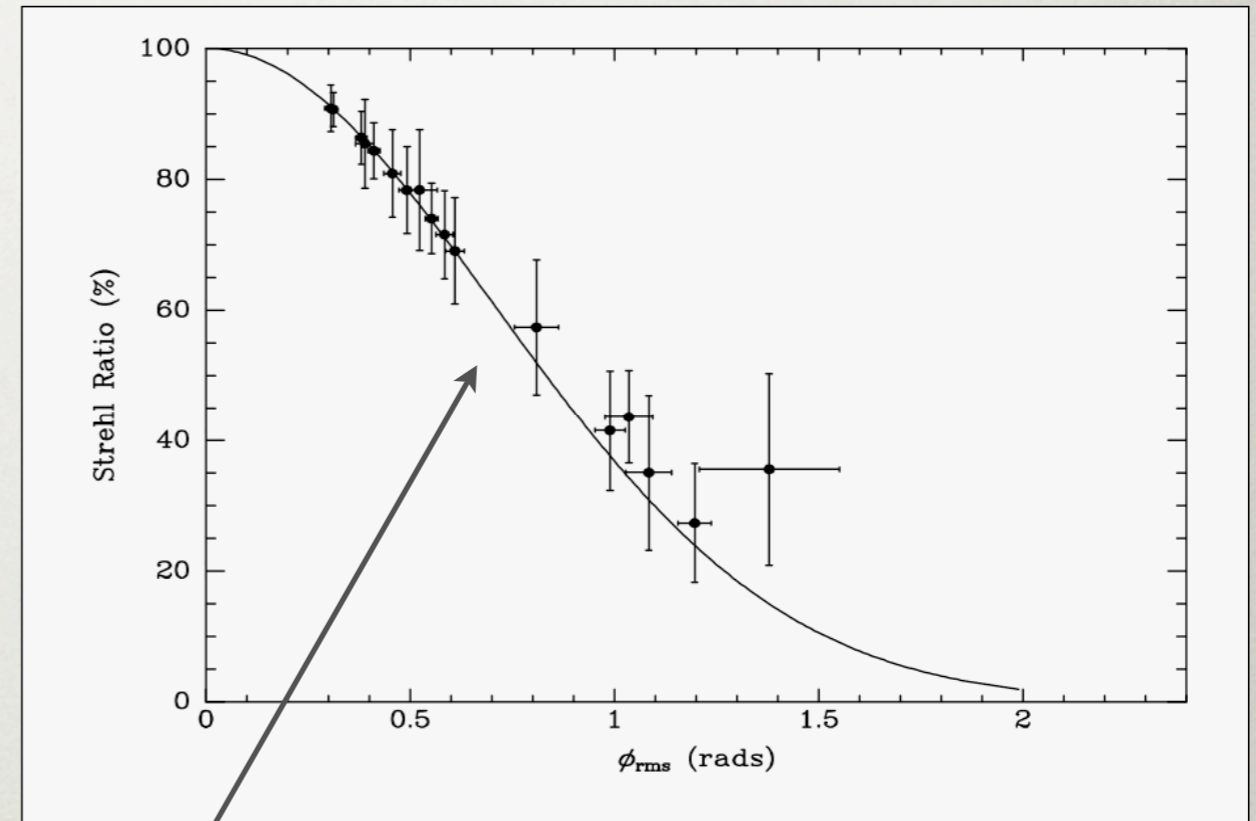
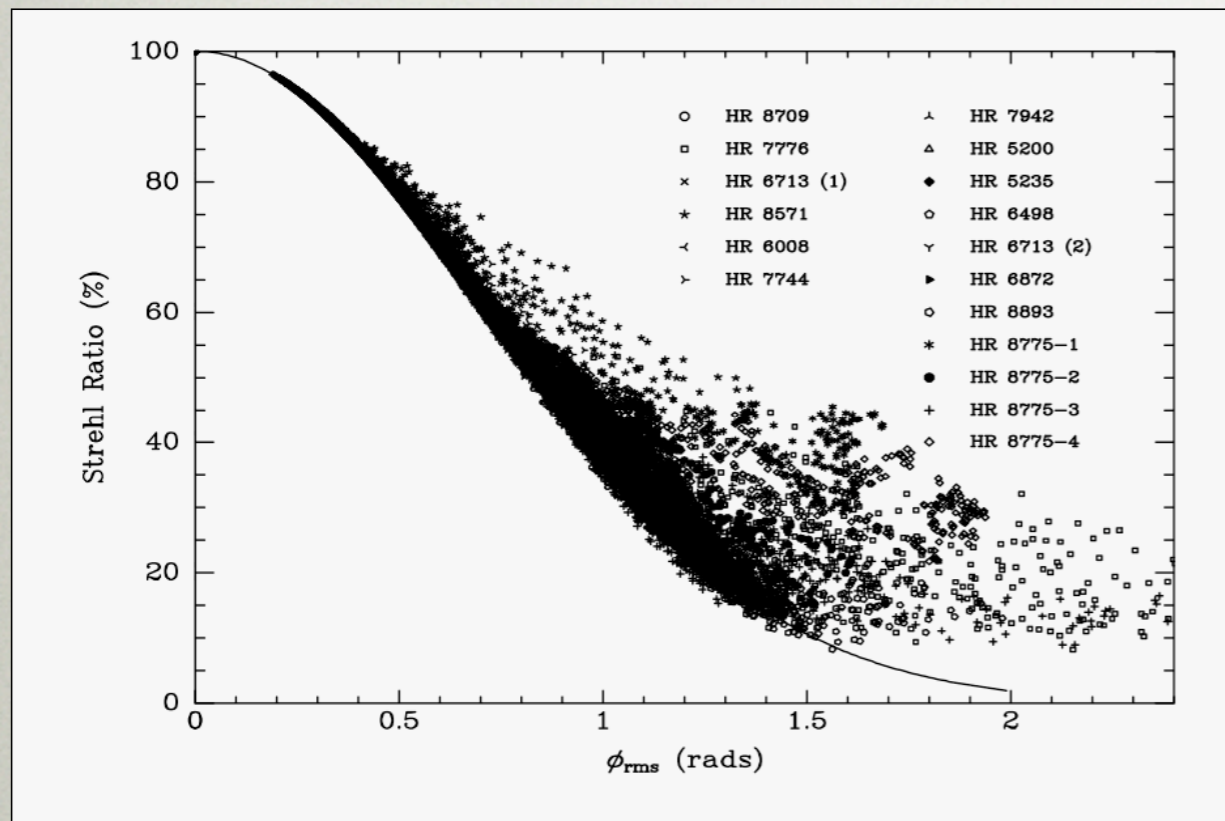


HR7942

HR5200

HR7776

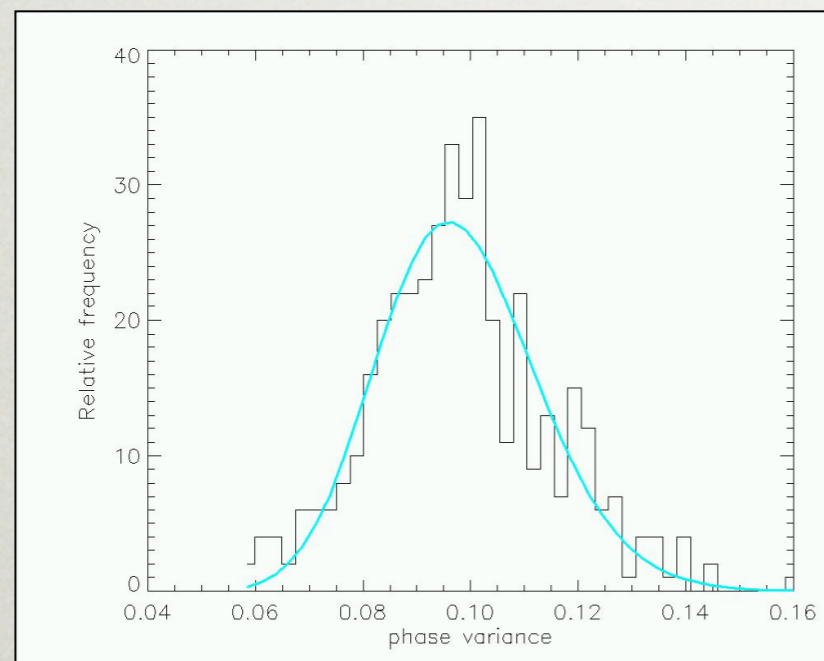
SR vs. RMS PHASE ERROR



EXTENDED MARECHAL APPROXIMATION

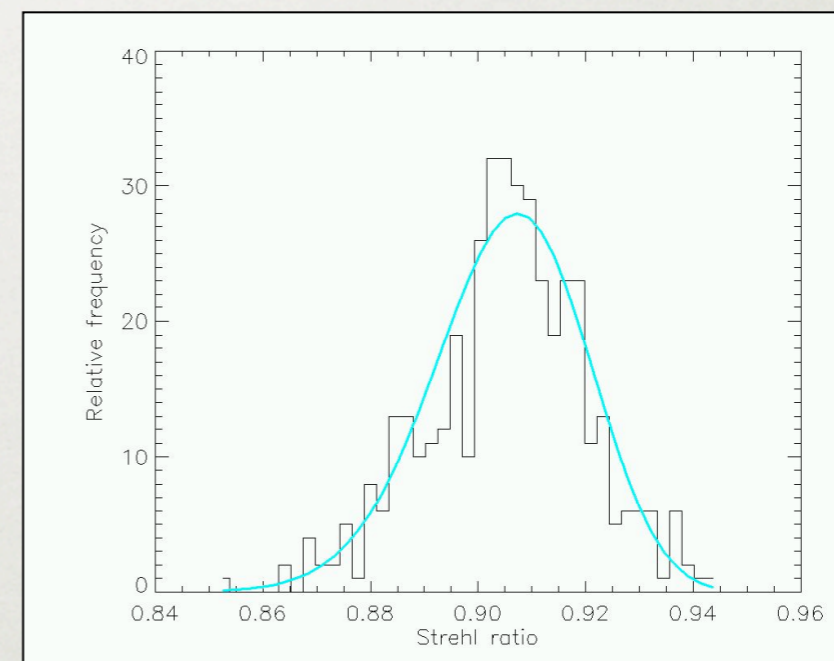
MODELS BY GLADYSZ ETAL.

$$f_{\text{var}} = \text{Gamma}\left(\frac{m-1}{2}, \frac{2\sigma^2}{m}\right)$$



GAMMA DISTRIBUTION FOR THE PHASE VARIANCE

$$f_S(s) = \frac{f_{\text{var}}(-\ln s)}{s}$$



DISTRIBUTION FOR THE INTENSITY

MODELS FOR THE SR STATISTICS

GAMMA DISTRIBUTION FOR THE
NEGATIVE COMPLEX AMPLITUDE

—————→
SOUMMER & FERRARI

DISTRIBUTION FOR THE INTENSITY

GAMMA DISTRIBUTION FOR
THE PHASE VARIANCE

—————→
GLADYSZ ETAL.

DISTRIBUTION FOR THE INTENSITY

THE LINK BETWEEN THE DISTRIBUTION FOR THE COMPLEX AMPLITUDE AND
PHASE VARIANCE IS OBVIOUS UNDER THE MARECHAL APPROXIMATION

$$I = e^{-\sigma_{\phi}^2}$$

$$\sqrt{I} = 1 - \frac{\sigma_{\phi}^2}{2}$$

CORONAGRAPHIC IMAGES

QUASI~STATIC SPECKLES

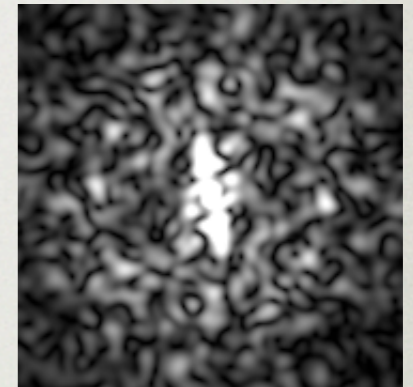
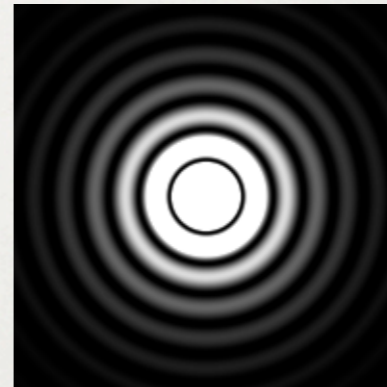
EFFECT OF A CORONAGRAPH

- THE DIRECT FOCAL PLANE AMPLITUDE IS:

$$\Psi_2(\mathbf{r}) = C_d(\mathbf{r}) + S(\mathbf{r})$$

STATIC
DIRECT
RESPONSE

SPECKLE
TERM



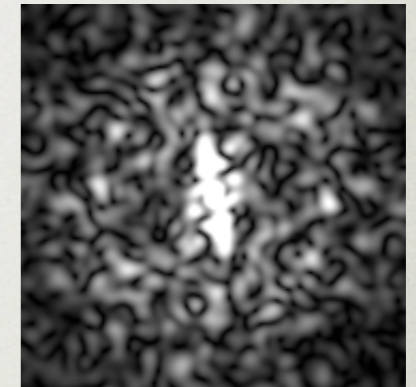
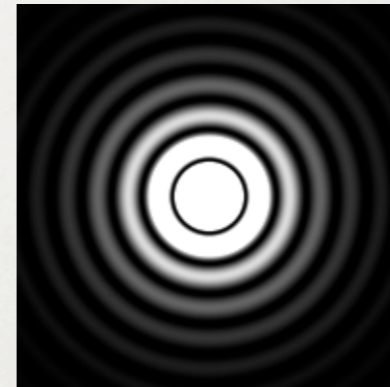
EFFECT OF A CORONAGRAPH

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$$\Psi_2(\mathbf{r}) = C_d(\mathbf{r}) + S(\mathbf{r})$$

STATIC
DIRECT
RESPONSE

SPECKLE
TERM



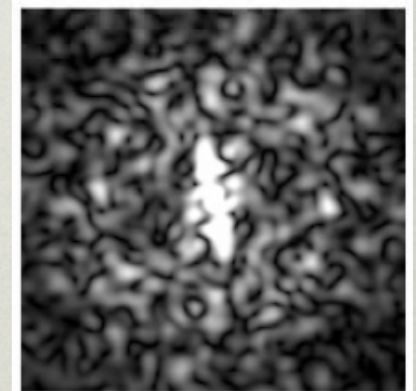
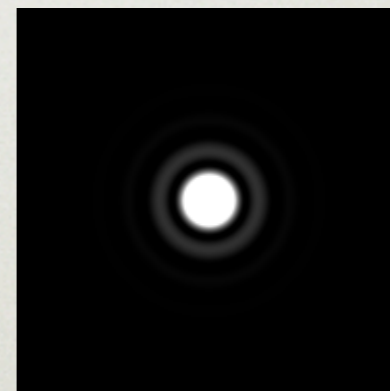
- THE CORONAGRAPHIC FOCAL PLANE AMPLITUDE IS:

$$\Psi_4(\mathbf{r}) = C_c(\mathbf{r}) + S(\mathbf{r}) - (S(\mathbf{r})M(\mathbf{r})) * \mathcal{F}[P'(\mathbf{r})]$$

$$\Psi_4(\mathbf{r}) = C_c(\mathbf{r}) + S(\mathbf{r})$$

STATIC
CORONAGRAPH
RESPONSE

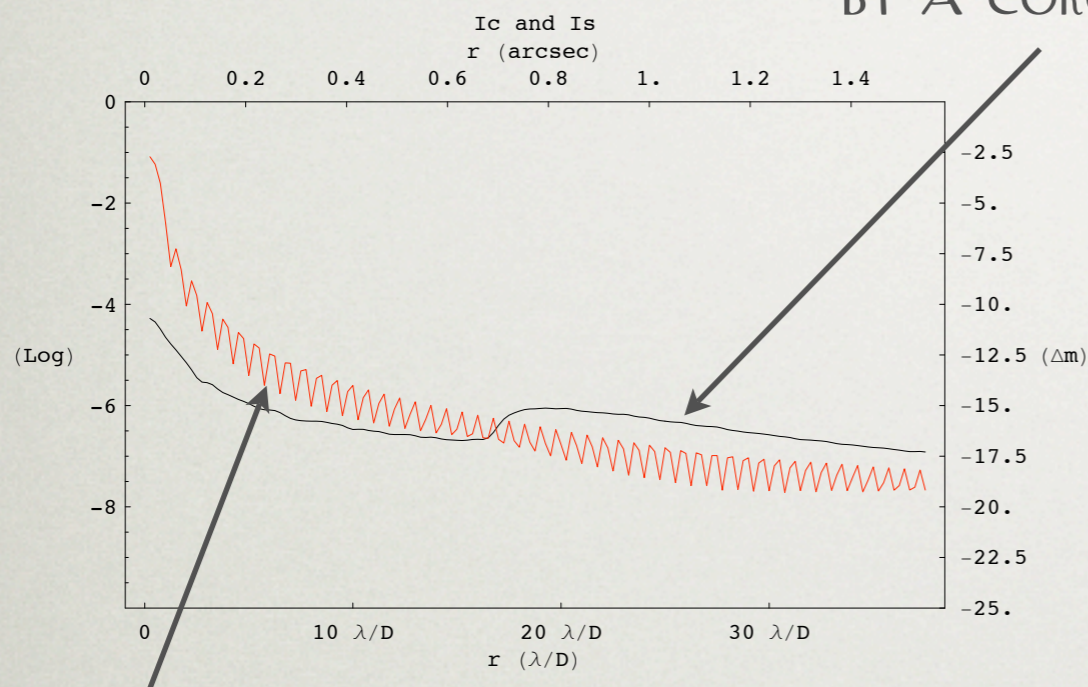
SPECKLE
TERM



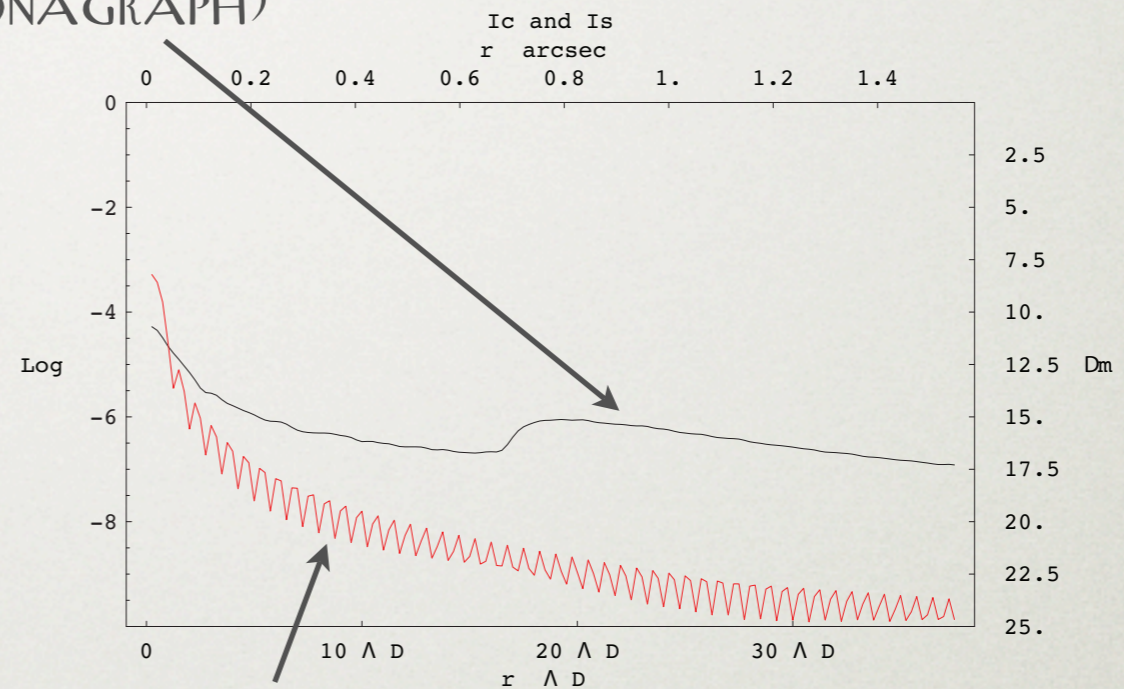
A CORONAGRAPH HAS A NO EFFECT ON THE SPECKLE COMPLEX AMPLITUDE TERM OUTSIDE THE MASK AREA

EFFECT OF A CORONAGRAPH

SPECKLE TERM: (NOT AFFECTED BY A CORONAGRAPH)



I_c WITHOUT CORONAGRAPH



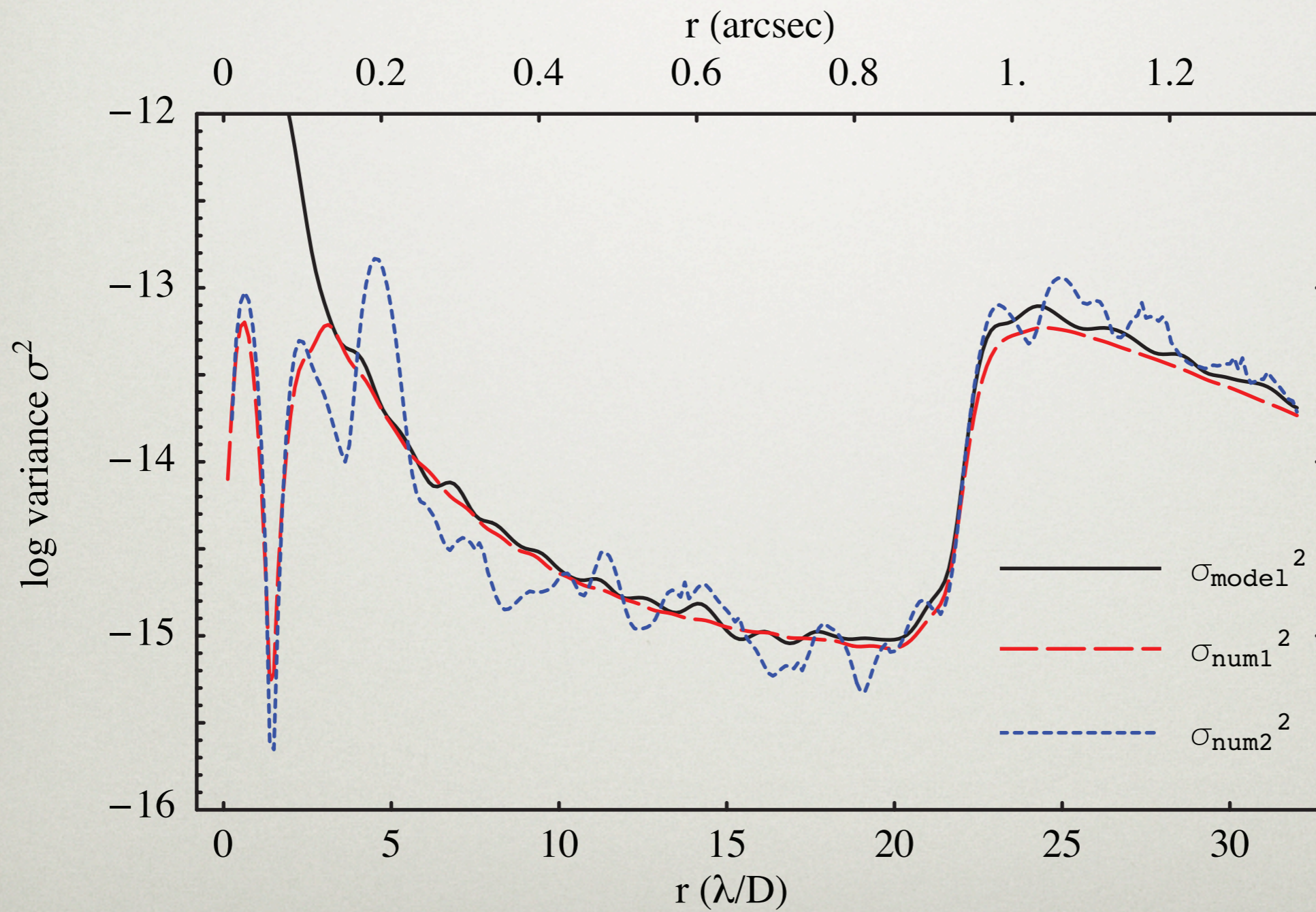
I_c WITH CORONAGRAPH

$$\sigma^2 = (2I_s I_c + I_c) + (I_s^2 + I_s) = \sigma_c^2 + \sigma_s^2$$

CAN BE REMOVED BY A CORONAGRAPH

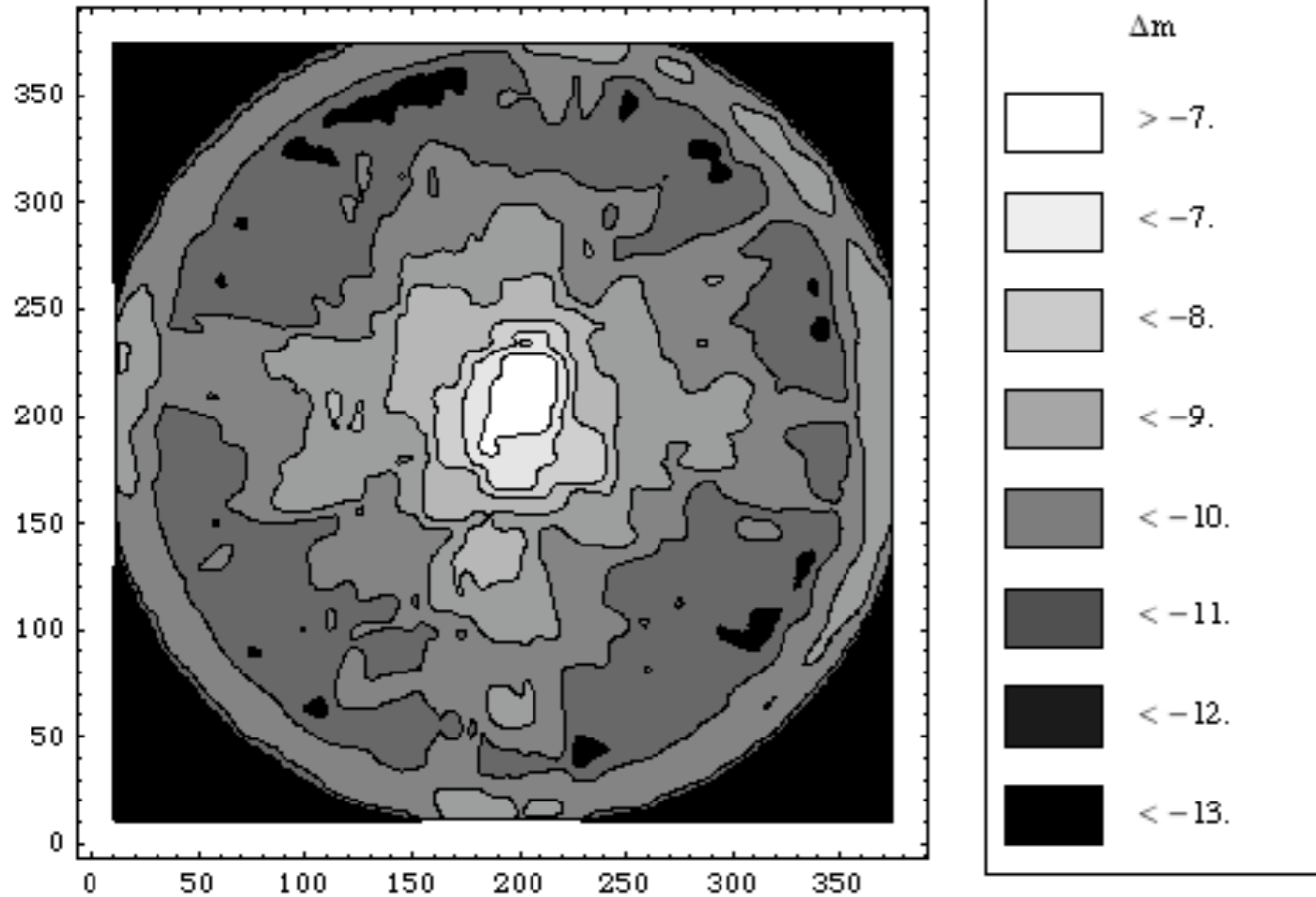
UNAFFECTED BY A CORONAGRAPH

MODEL VS. NUMERICAL SIMULATION

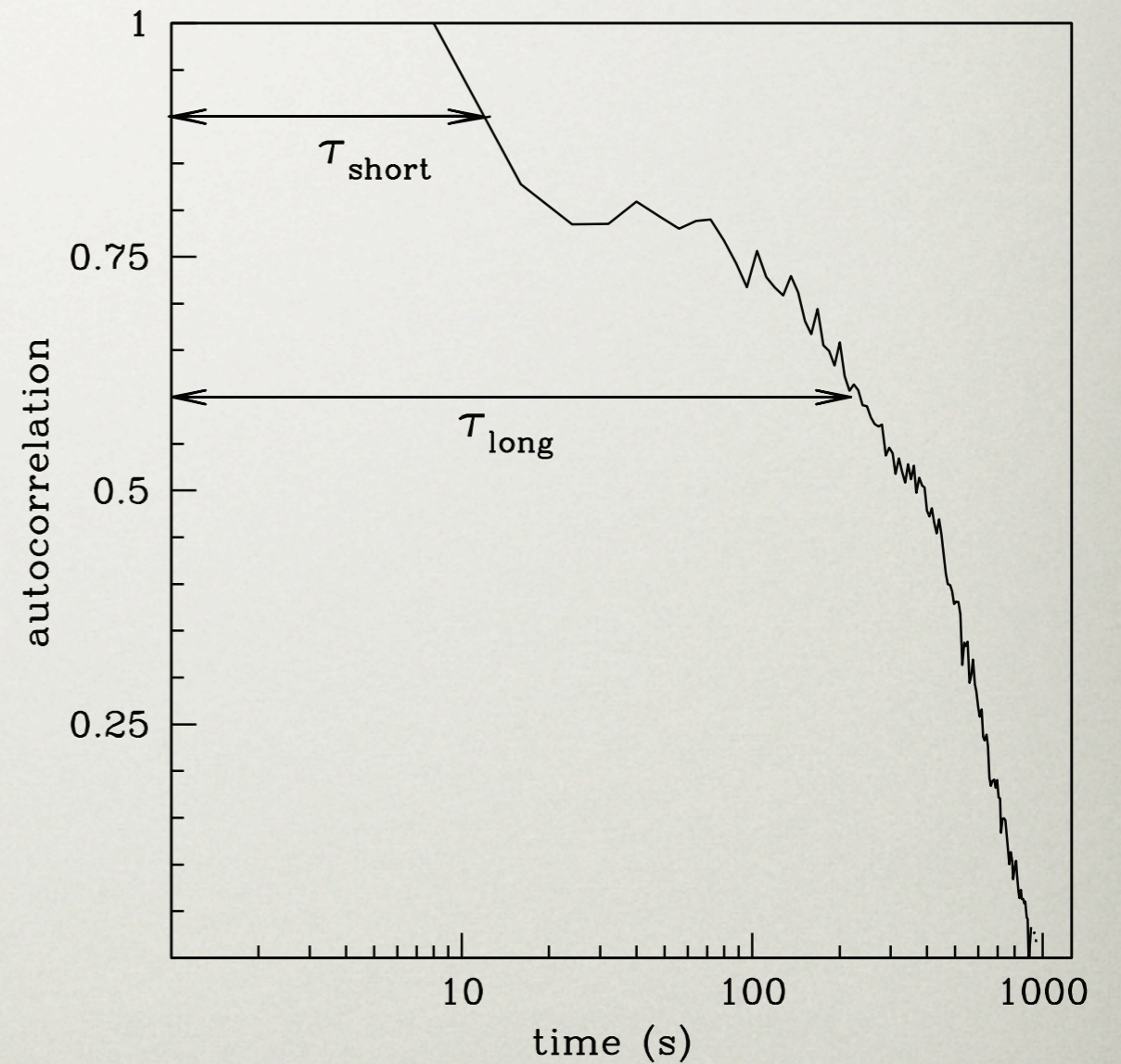


TEMPORAL PROPERTIES OF SPECKLES

Vega dynamic range, 5σ , 008s



Soummer et al 2006



Hinlkey et al 2006

STATIC + QUASI~STATIC

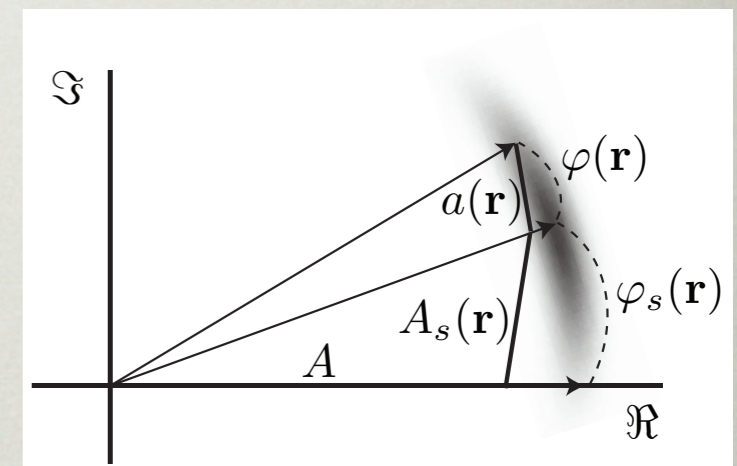
- QUASI~STATIC SPECKLES CAN ALSO BE INCLUDED
- DECOMPOSITION INTO: PERFECT, STATIC, QUASI STATIC AND ATMOSPHERIC TERMS:

$$\Psi_1 = A + A_s(x) + a_1(x) + a_2(x)$$

$$\Psi_4 = \tilde{C} + S_1(x) + S_2(x)$$

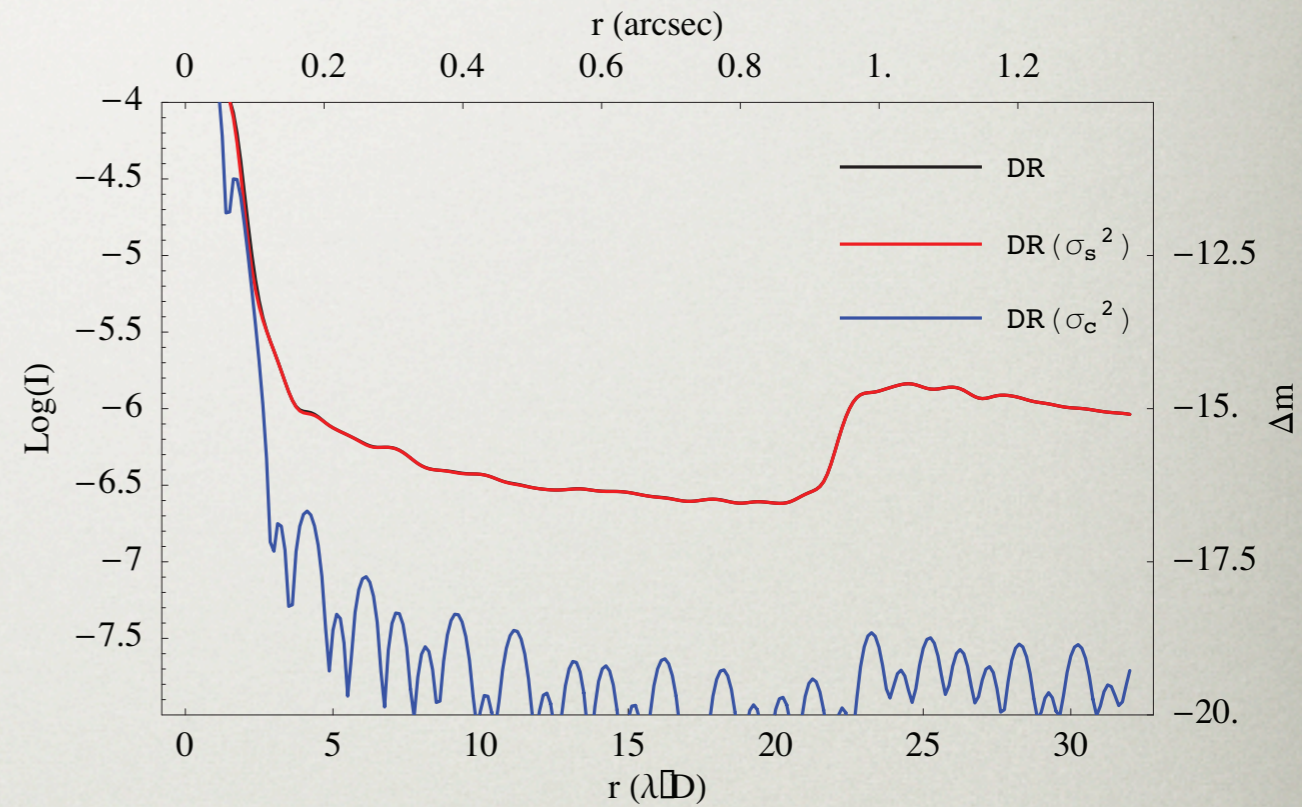
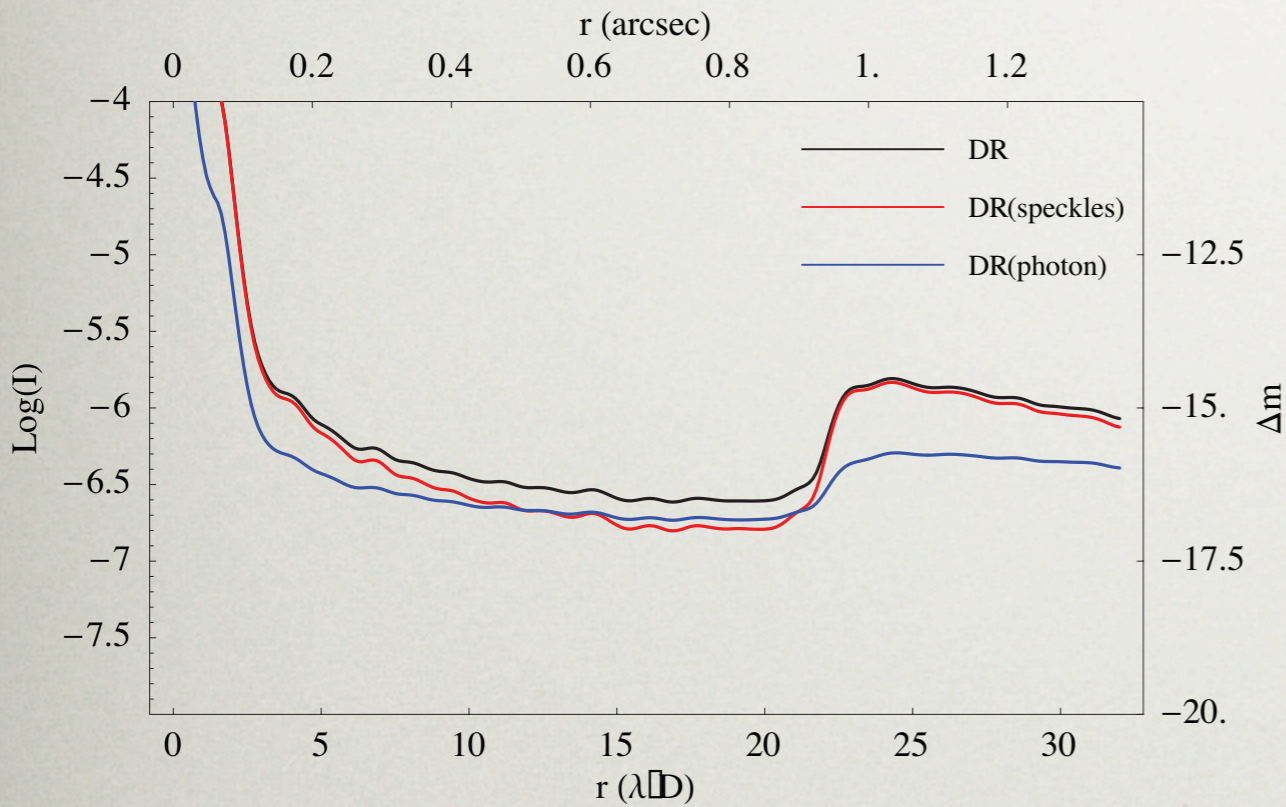
FAST
SPECKLES

SLOW
SPECKLES

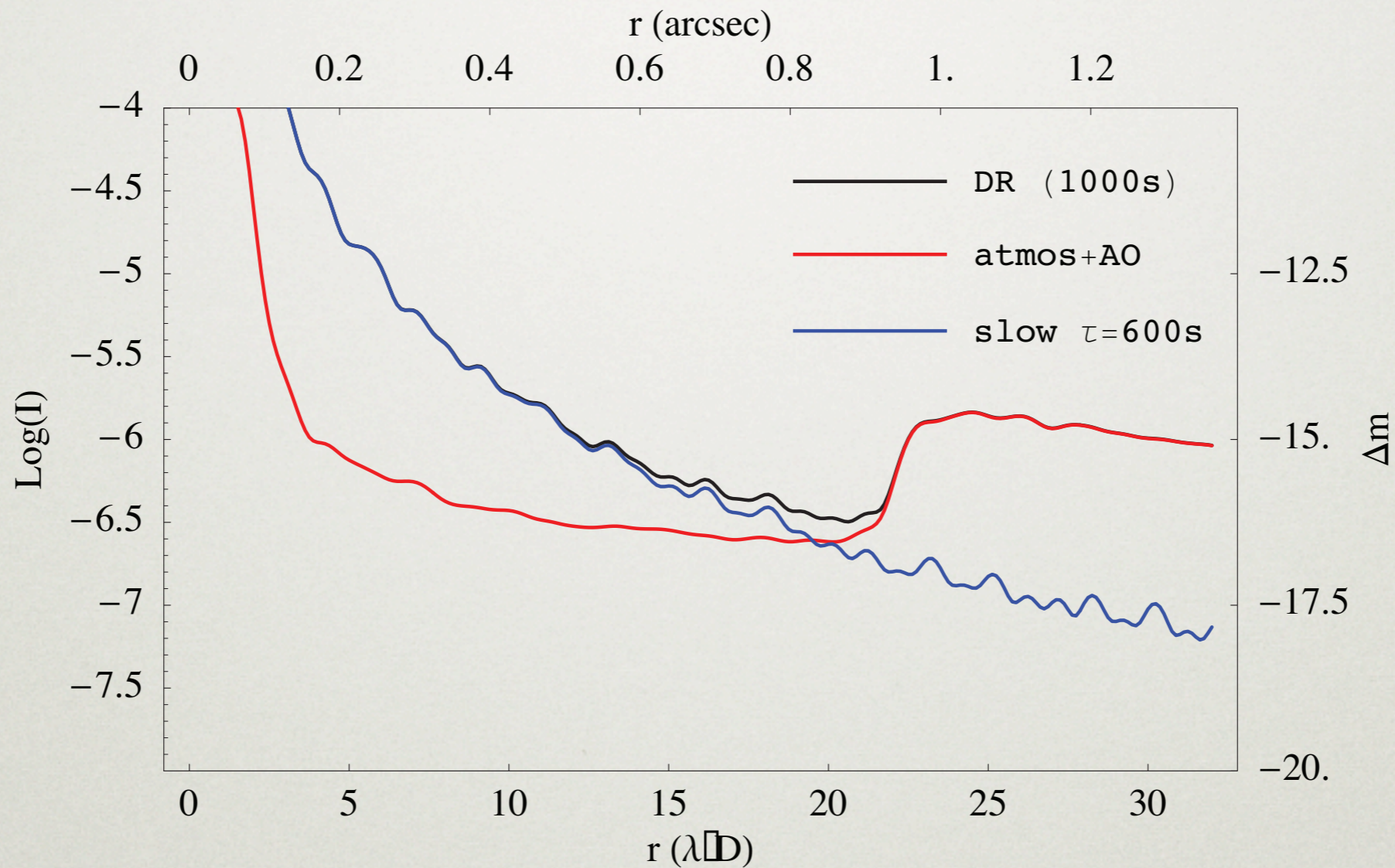


$$\sigma_I^2 = N(I_{s1}^2 + NI_{s2}^2 + 2I_c(I_{s1} + NI_{s2}) + 2I_{s1}I_{s2}) + \sigma_p^2$$

DYNAMIC RANGE



QUASI-STATIC SPECKLES



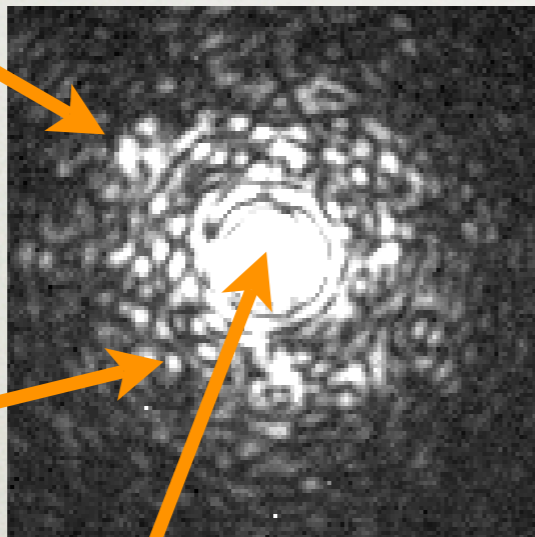
SUMMARY

CORONAGRAPH REDUCES
BOTH SPECKLE AND PHOTON
NOISE BY REMOVING THE
SPECKLE PINNING

EXPONENTIAL DISTRIBUTION FOR
PERFECT CORONAGRAPH

PINNING BY CONSTANT
PSF (INCLUDING
STATIC ABERRATIONS)

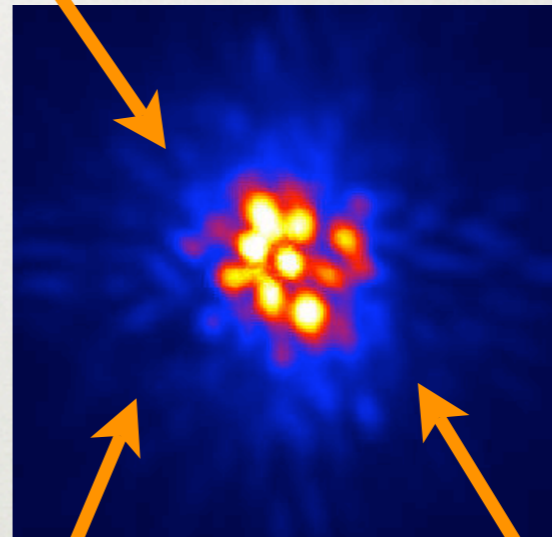
DIRECT



RICIAN DISTRIBUTION,
VERIFIED ON LICK AND
PALOMAR DATA

NEGATIVELY SKEWED DISTRIBUTION AT HIGH SR
VERIFIED ON LICK AND AEOS DATA
(RICIAN STATISTICS AT VERY LOW CORRECTION)

CORONAGRAPHIC



PINNING BY STATIC
ABERRATIONS NOT
FILTERED BY THE
CORONAGRAPH

ANALYTICAL VARIANCE
EXPRESSION WITH FAST
AND QUASI-STATIC
ABERRATIONS

CONCLUSIONS

$$\sigma_I^2 = N(I_{s1}^2) + NI_{s2}^2 + 2I_c(I_{s1} + NI_{s2}) + 2I_{s1}I_{s2} + \sigma_p^2$$

Diagram annotations:

- AO SYSTEM** (purple text) with arrows pointing to I_{s1}^2 and $2I_c(I_{s1} + NI_{s2})$.
- CAN BE REMOVED BY A CORONAGRAPH** (orange text) with an arrow pointing to $2I_c(I_{s1} + NI_{s2})$.
- AFFECTED BY CAL SYSTEM (SPECKLE NULLING)** (black text) with an arrow pointing to $2I_{s1}I_{s2}$.

REMAINING PARTS REMOVED BY POST PROCESSING

(ADI, CHROMATIC/POLARIMETRIC SPECKLE SUPPRESSION ETC.)