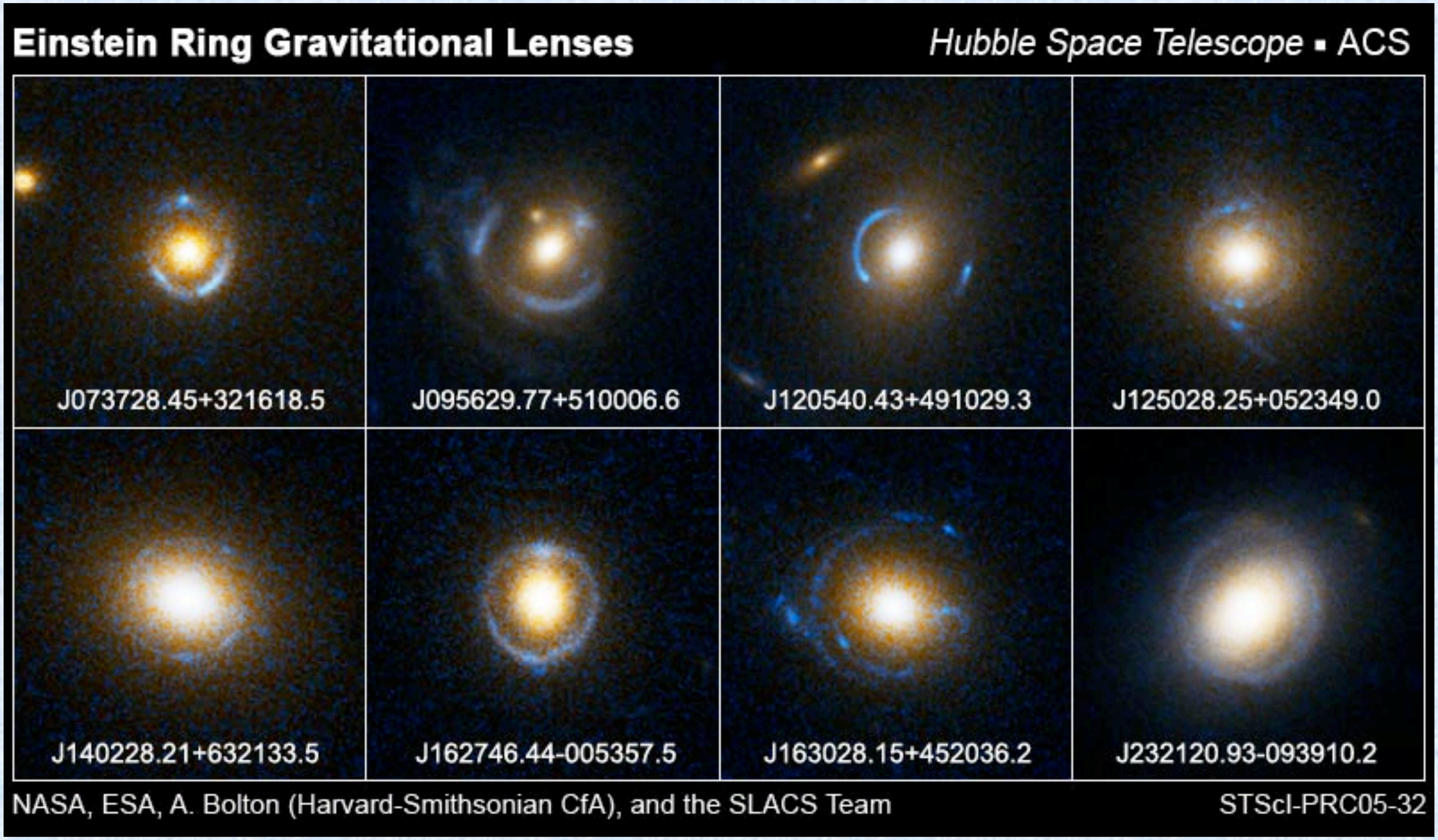
The kinematics of Einstein Rings

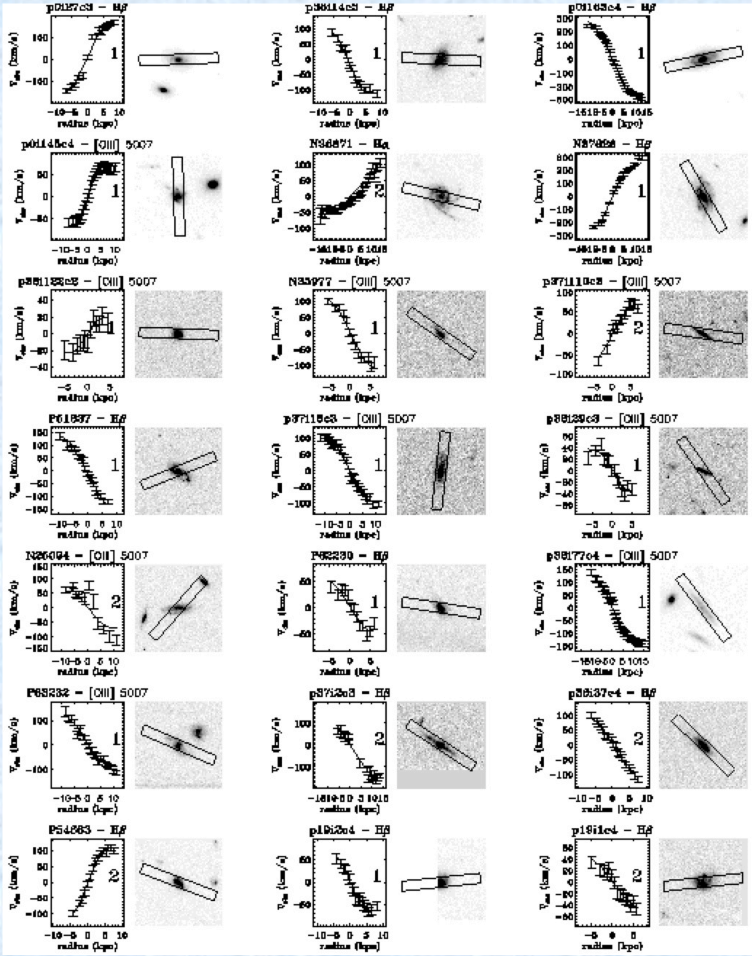


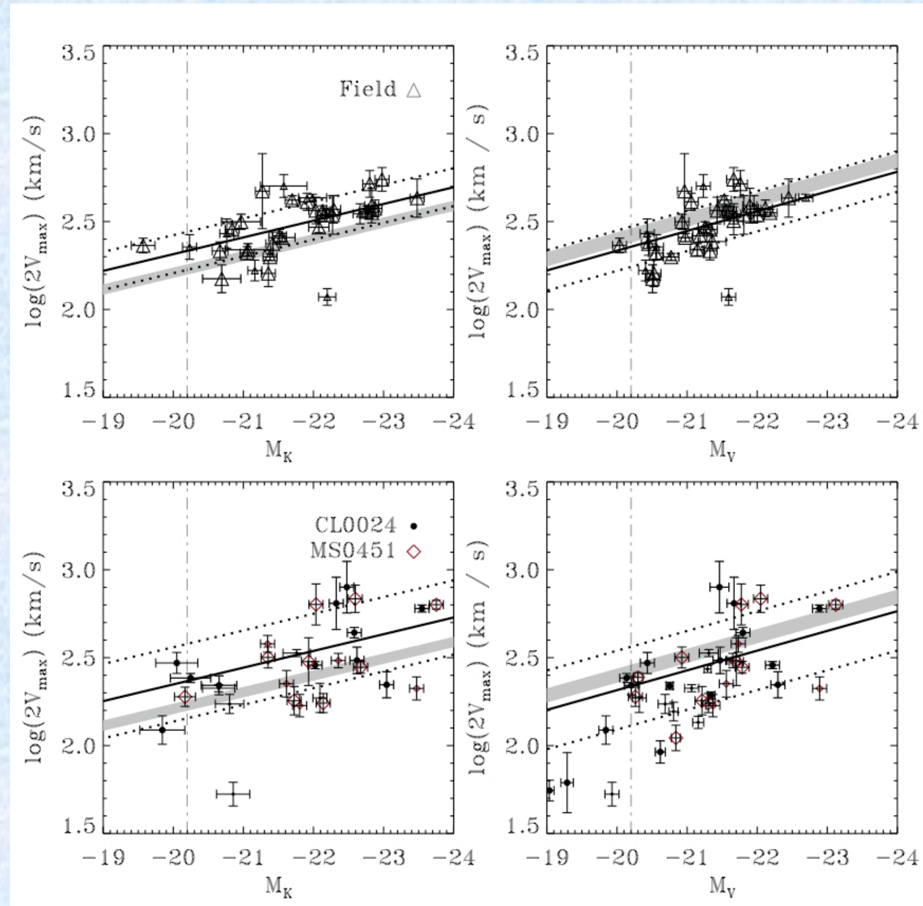
Tommaso Treu, Phil Marshall and Laura Melling (UCSB)

Why do we care?

- Lensing measures mass:
  - **Exploiting lensing achromaticity improves knowledge of gravitational potential of deflector**
- Lensing magnifies, hence “gravitational telescopes”:
  - The internal structure of distant galaxies can be study with a typical factor of 10 improvement in sensitivity and spatial resolution

Example application: Tully Fisher to Vmax<100 km/s

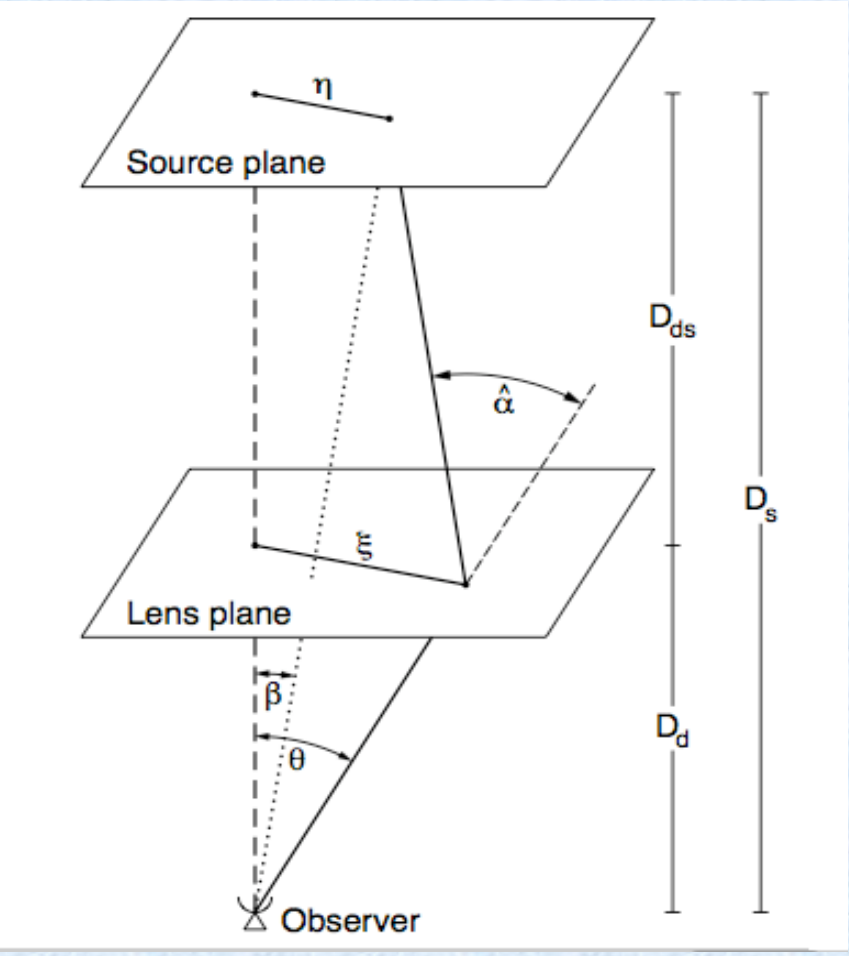




Moran, Miller, TT, Ellis & Smith 2007; and many more!!

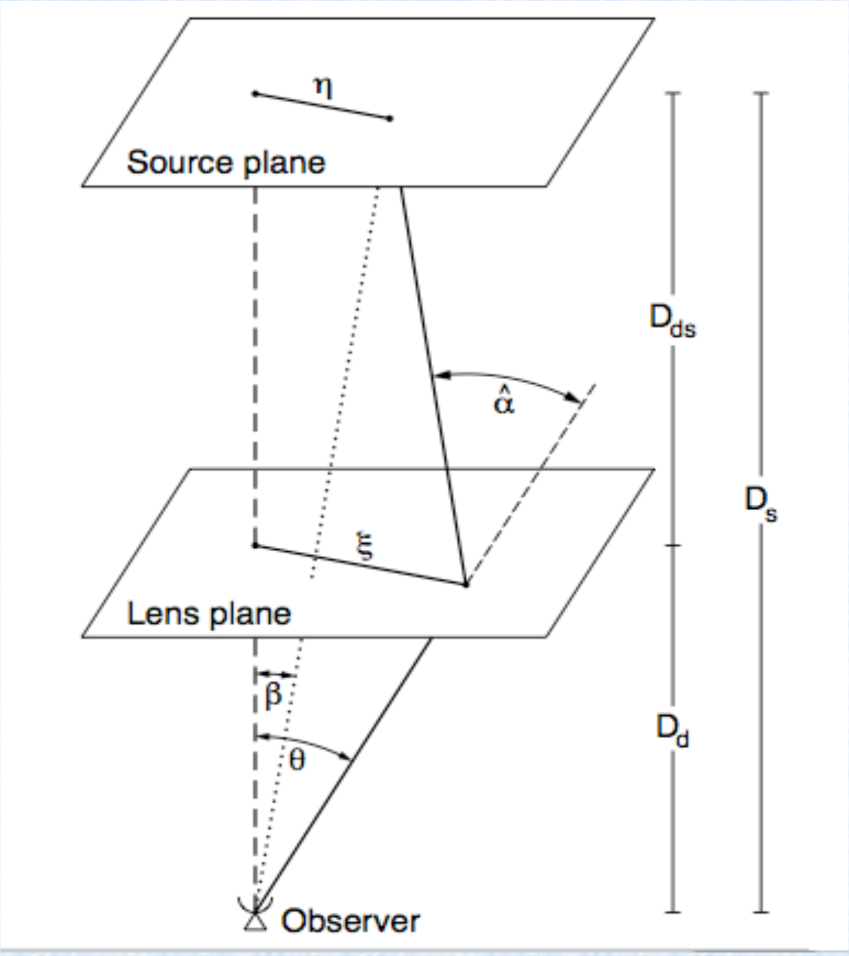
Strong Lensing Basics. I:

- Strong lensing can be seen as a mapping from the source plane (what would be seen without a lens) to the image plane (what is actually seen)
- The transformation is the so called lens-equation:
- $\theta=\beta+\alpha(\theta)$

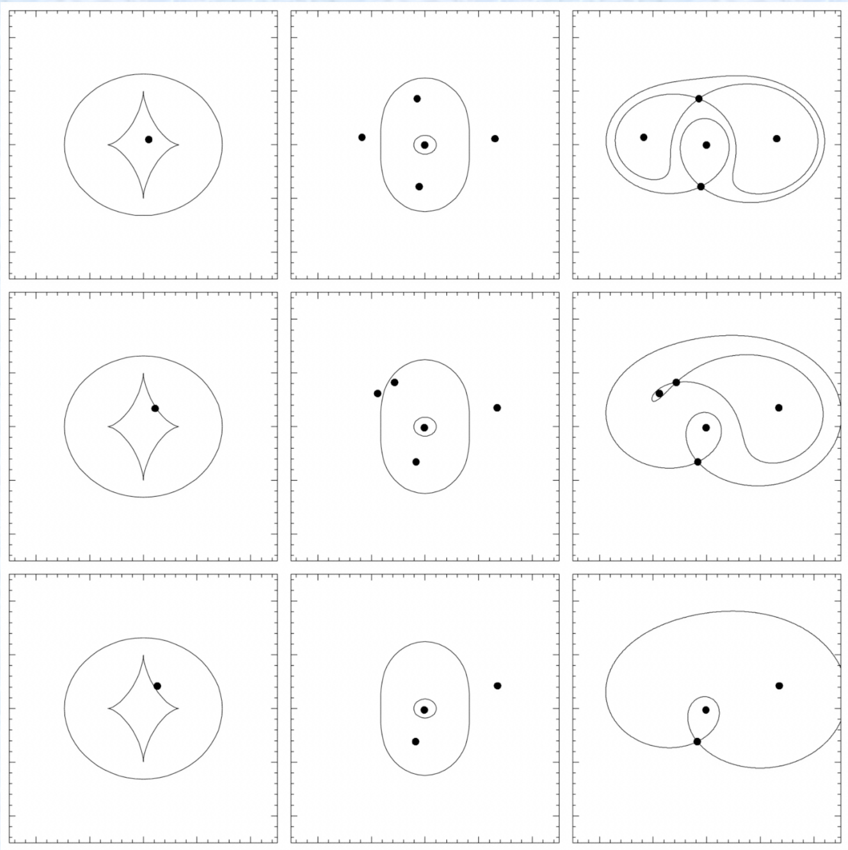


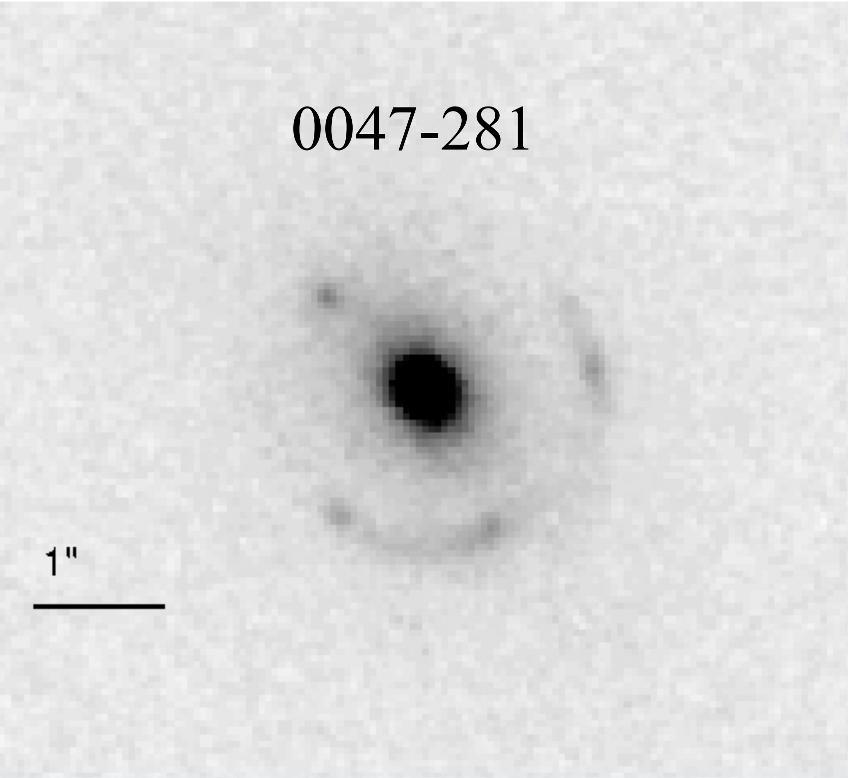
Strong Lensing Basics. II:

- Deflection angle α is gradient of gravitational potential: lensing measures mass.
- Lensing preserves surface brightness: magnification is given by the Jacobian of the transformation



Strong Lensing Basics. III:





Koopmans & Treu 2003

Curves where the transformation is singular are called caustics and critical lines. They correspond to infinite magnification. Sources get multipled images when they are inside caustics. They are typically highly magnified.

Properties of lens mapping:

- Non-linear
- Preserves surface brightness
- **Independent of frequency (ACHROMATIC)**
- Magnifies sources

Kinematic-lensing

- Traditional lensing exploits preservation of surface brightness to construct a model of the potential of the lens and the surface brightness of the source (e.g. Marshall's talk)
- **Kinematic lensing exploit lensing achromaticity:**
  - **Improve model of lens potential**
  - **Reconstruct and super-resolve the velocity field of a magnified source**
  - **Can be done with emission lines of background source and therefore simplifies lens subtraction**

Lens System Modeling

Use parametric forms:

$\Sigma(\theta; \boldsymbol{m})$	Lens mass
$SB^{SP}(\beta; \vec{\boldsymbol{s}})$	Source surface brightness
$v_z^{SP}(\beta; \vec{\boldsymbol{u}})$	Source line of sight velocity

<u>'Simplest' Example:</u>

SIE:	σ,	b/a,	PA,	x,	y	
Exponential disk:	M,	r_e	i,	PA,	x,	y,
Arctangent:	V_{max},	r_0,	i			

13 parameters

Generating 'Ideal' Image Plane

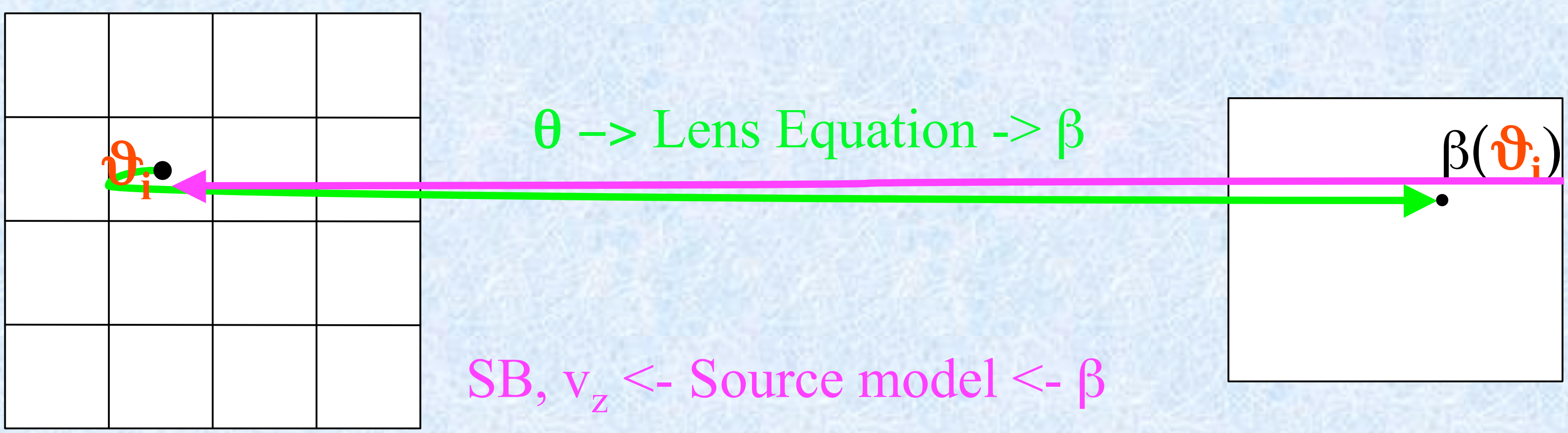
Pixellate image plane: $\{\vartheta_i\}$

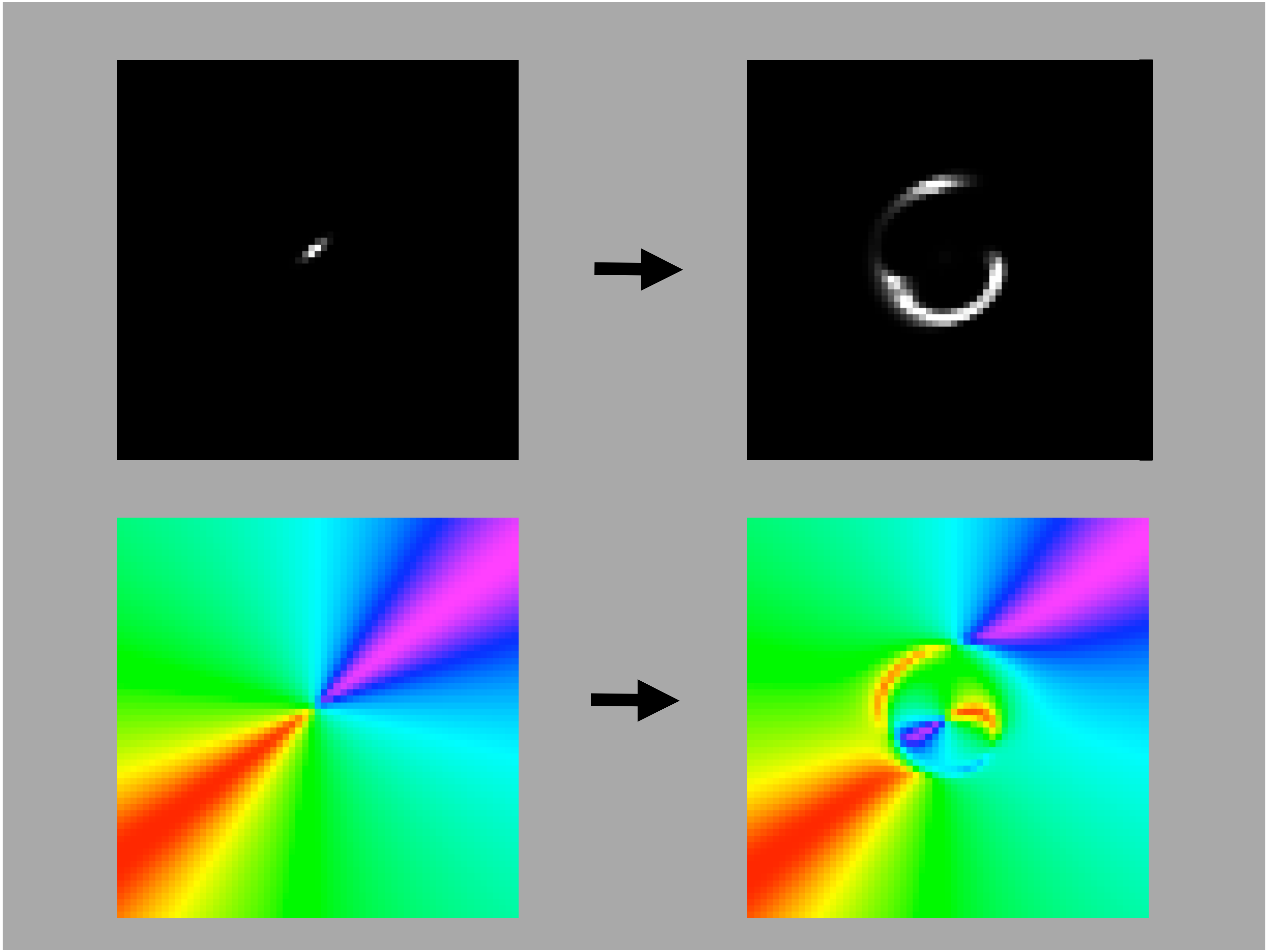
For each , use lens equation to calculate β.

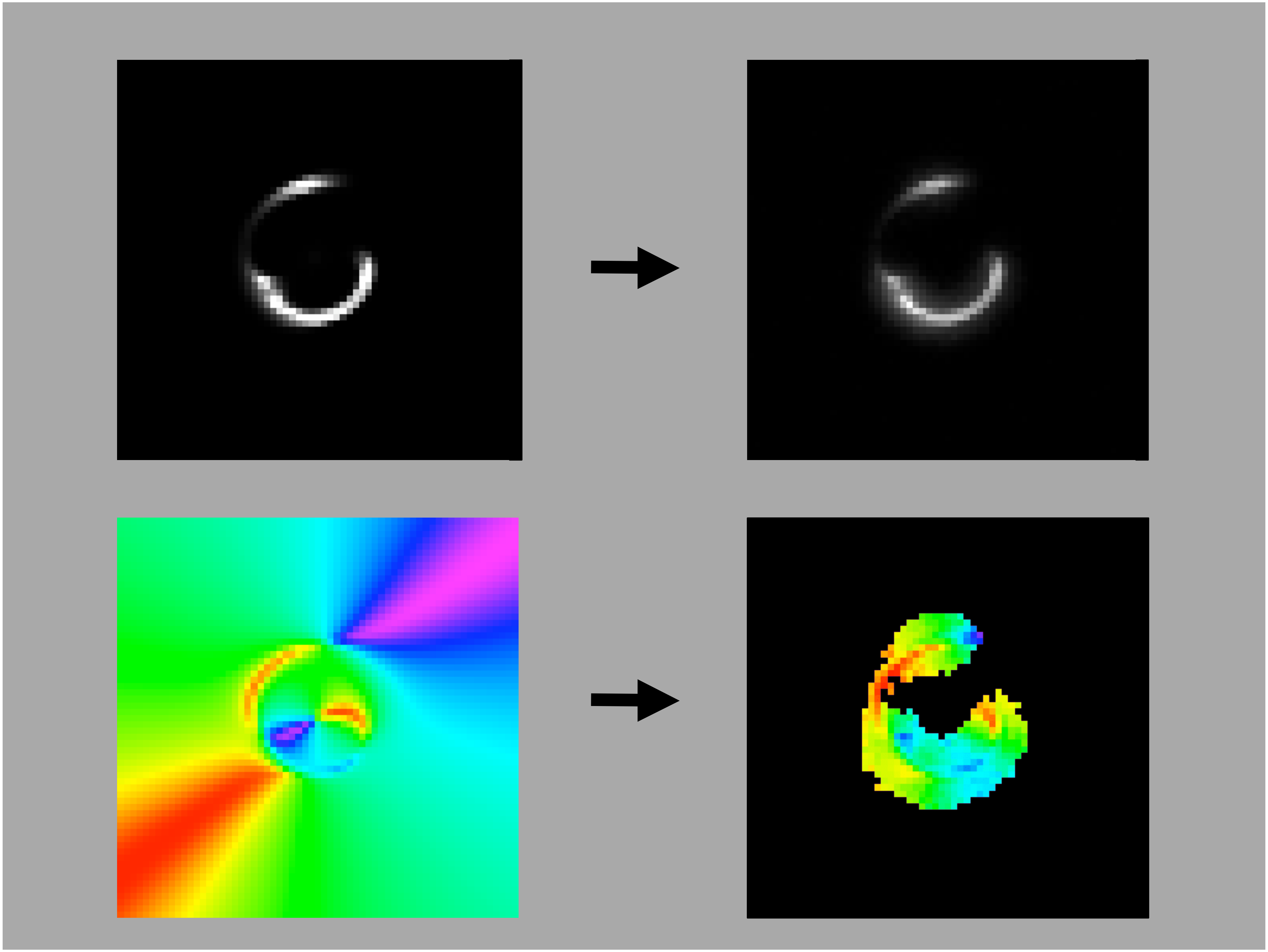
Map conserved quantities:

$$SB^{IP}(\vartheta_i; \vec{m}, \vec{s}) = SB^{SP}(\beta(\vartheta_i; \vec{m}); \vec{s})$$

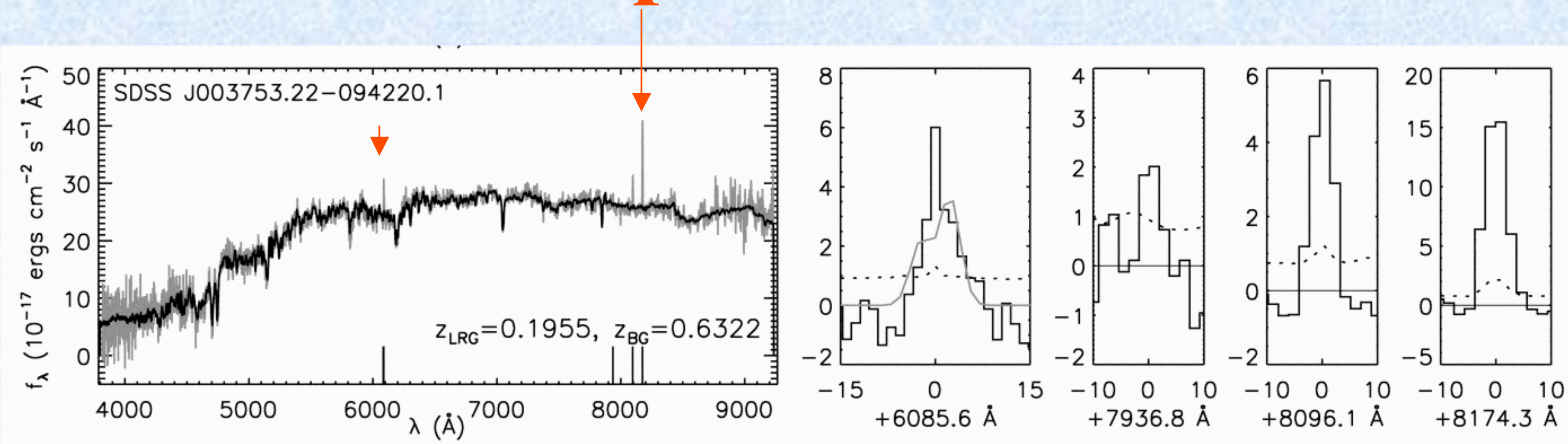
$$v_z^{IP}(\vartheta_i; \vec{m}, \vec{u}) = v_z^{SP}(\beta(\vartheta_i; \vec{m}); \vec{u})$$







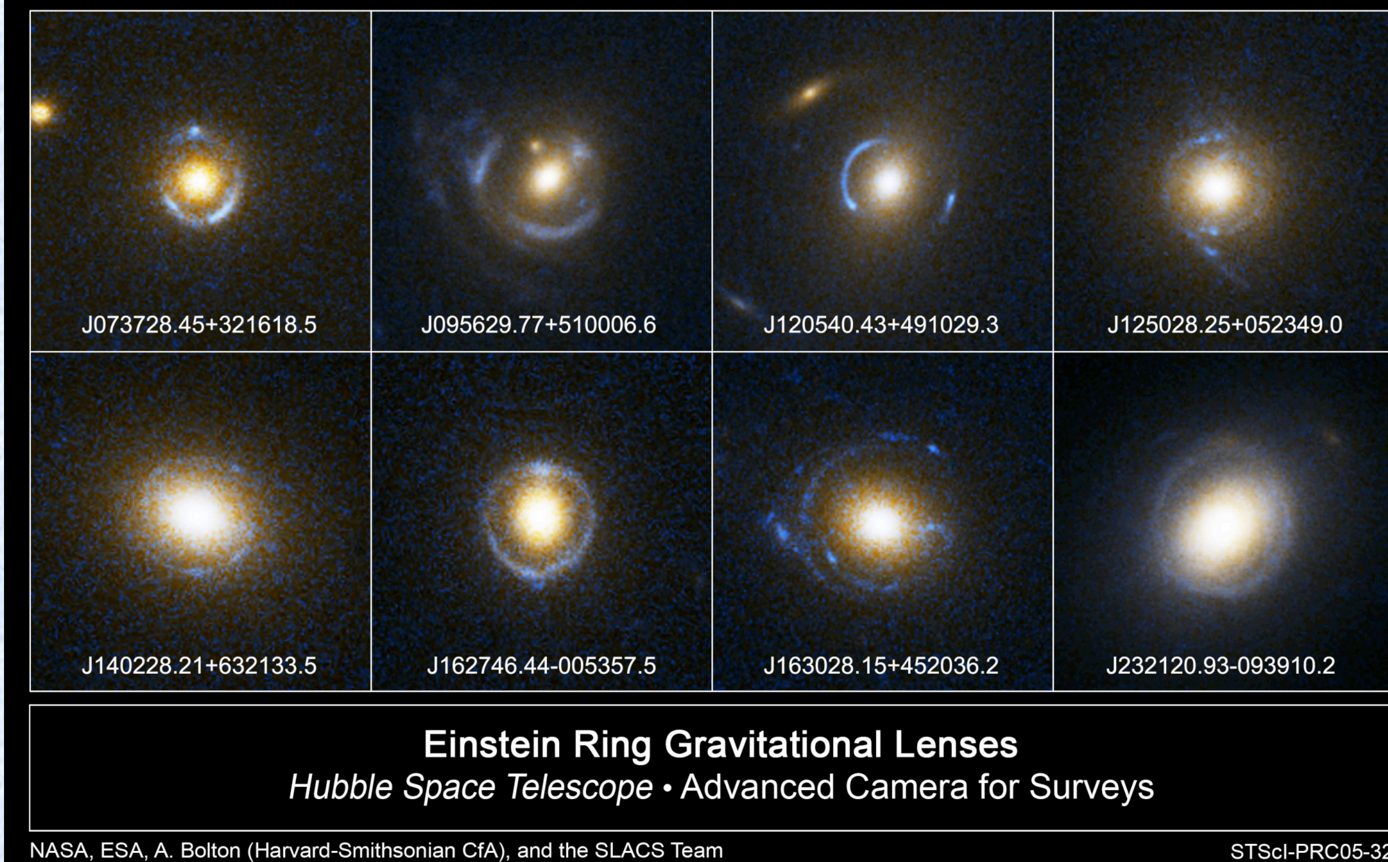
Can we do it in practice? Sample: SLACS



- Candidate lenses selected from SDSS as red galaxies with "spurious" emission lines (Bolton et al. 2004,2005,2006,2007)
- PROS: Huge sample (88 lenses so far!). Bright emission lines by construction. Know flux Hb -> Ha
- CONS: Source redshift < 0.8 because of selection. Ha @ Z/J

SLACS: examples

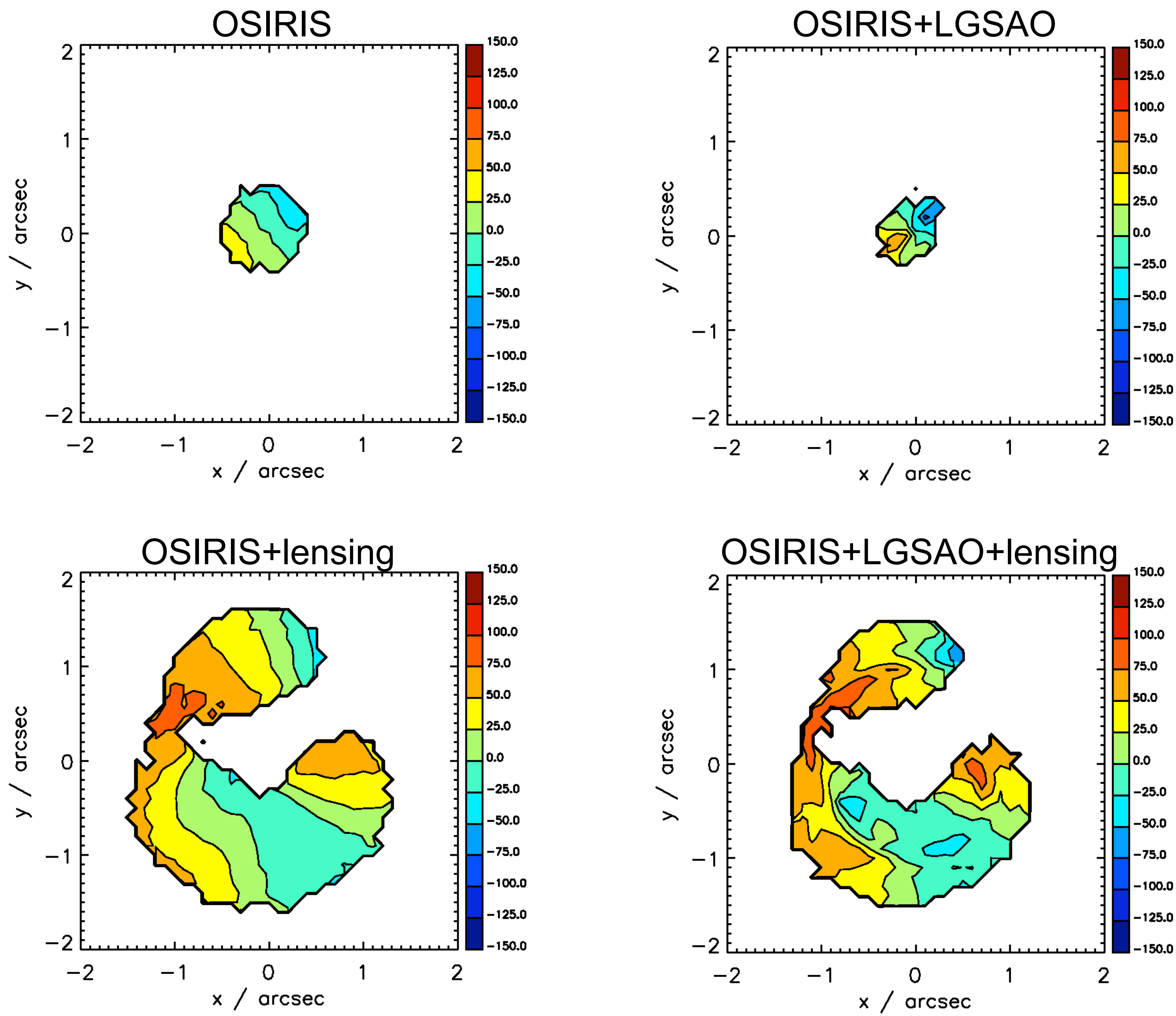
See www.slacs.org and Bolton et al. 2006, 2007

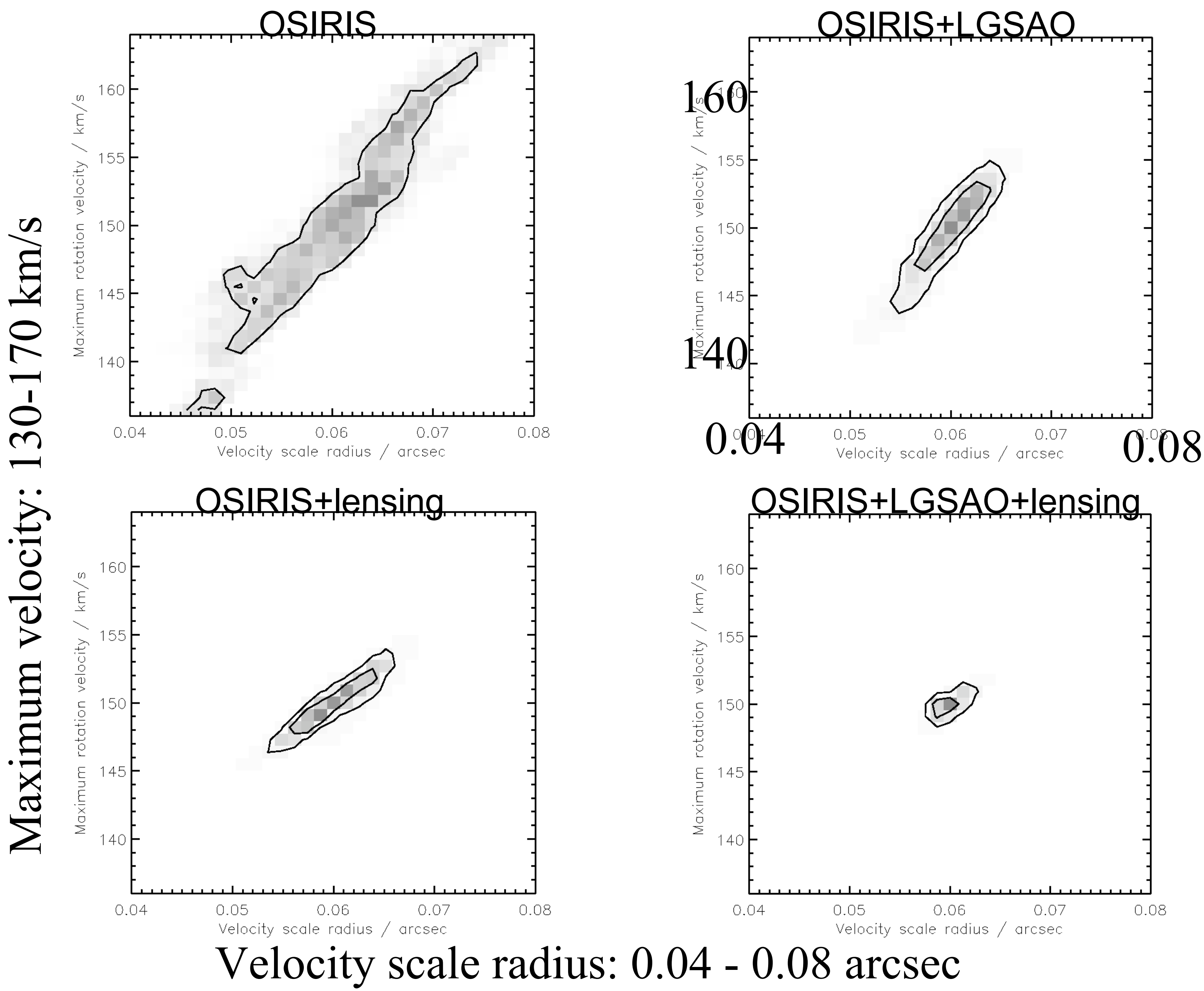


Example. I: velocity field

Test KLens on a SLACS lens:

1 . Using the ACS image, model the lens using surface brightness only.
2 . Add 'reasonable' velocity field parameters and generate synthetic OSIRIS image and velocity data.
3 . Reconstruct the source and estimate error on parameters.
    1 . Unlensed
    2 . .Unlensed w/ LGSAO
    3 . Lensed
    4 . Lensed w/ LGSAO
4 . Assuming: 9500s exptime; line flux 5e-16 cgs; Z-band; Strehl 0.2


OSIRIS
OSIRIS+LGSAO
OSIRIS+lensing
OSIRIS+LGSAO+lensing
x / arcsec
y / arcsec
150.0
125.0
100.0
75.0
50.0
25.0
0.0
−25.0
−50.0
−75.0
−100.0
−125.0
−150.0

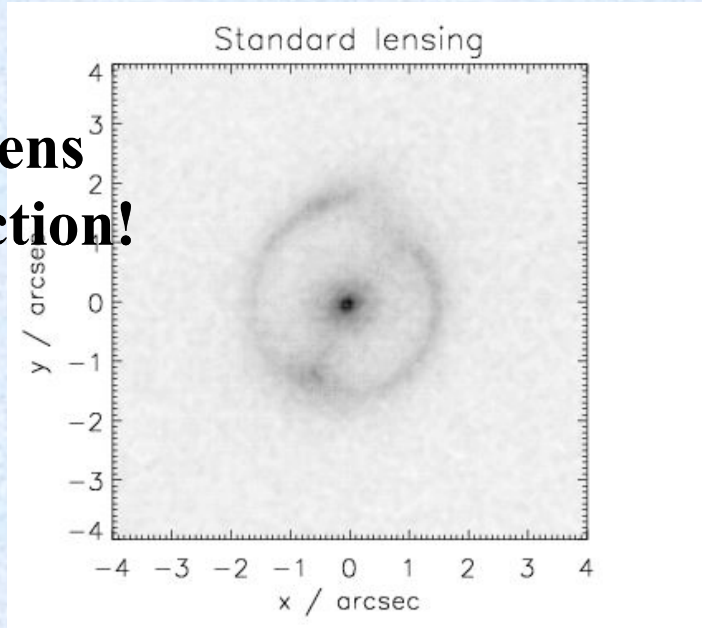

OSIRIS
OSIRIS+LGSAO
OSIRIS+lensing
OSIRIS+LGSAO+lensing
Maximum rotation velocity / km/s
Velocity scale radius / arcsec
140
145
150
155
160
0.04
0.05
0.06
0.07
0.08
Maximum velocity: 130-170 km/s
Velocity scale radius: 0.04 - 0.08 arcsec

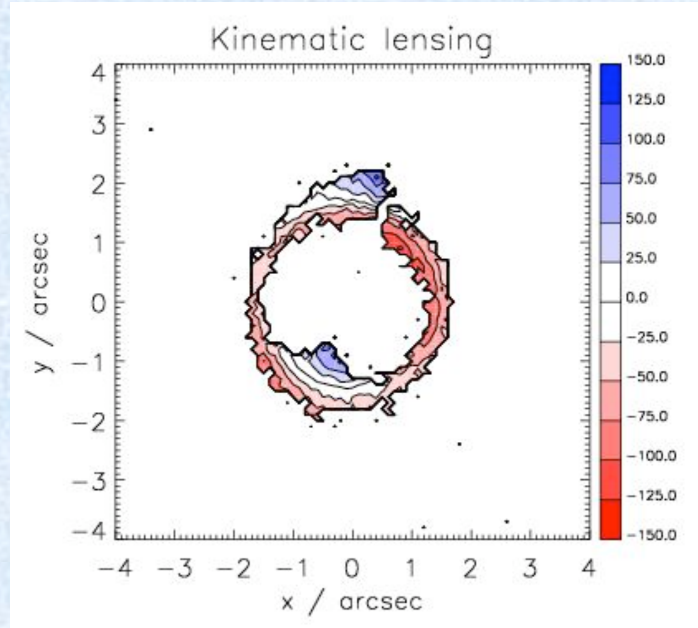
Example. II: lens model

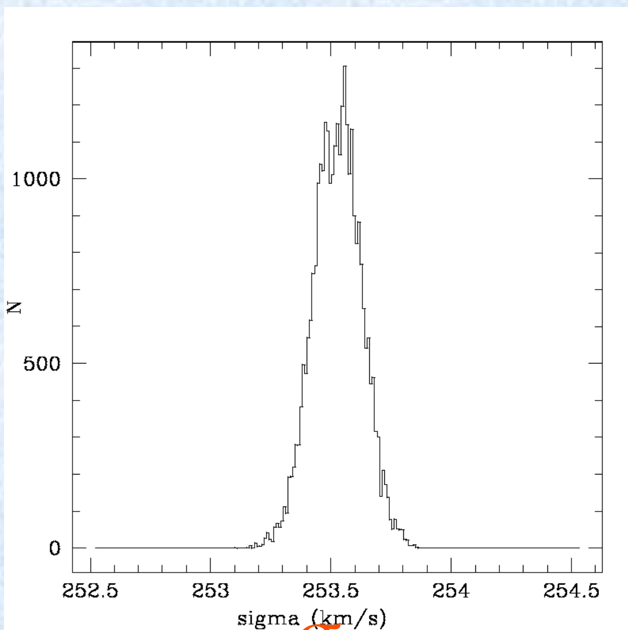
Simulated lens from SLACS:

1. Using the ACS image, model the lens using surface brightness only.
2. Add 'reasonable' velocity field parameters and generate synthetic OSIRIS image and velocity data.
3. Reconstruct the lens parameters for
    1. LGSAO+OSIRIS emission line imaging
    2. LGSAO+OSIRIS emission lines imaging + velocity field
4. Assuming: 10800s exptime; line flux 5e-16 cgs; Z-band; Strehl 0.15

Clean lens subtraction!

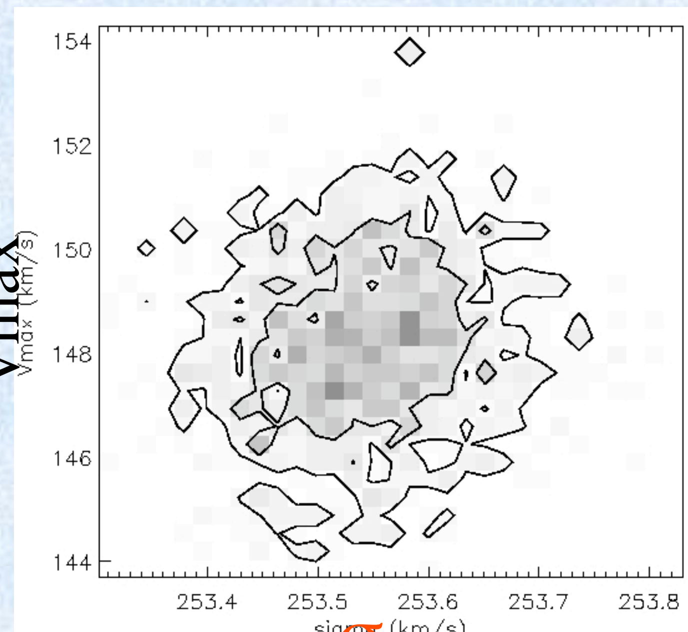






σ

Vmax



σ

Does it work in practice? It's hard!!

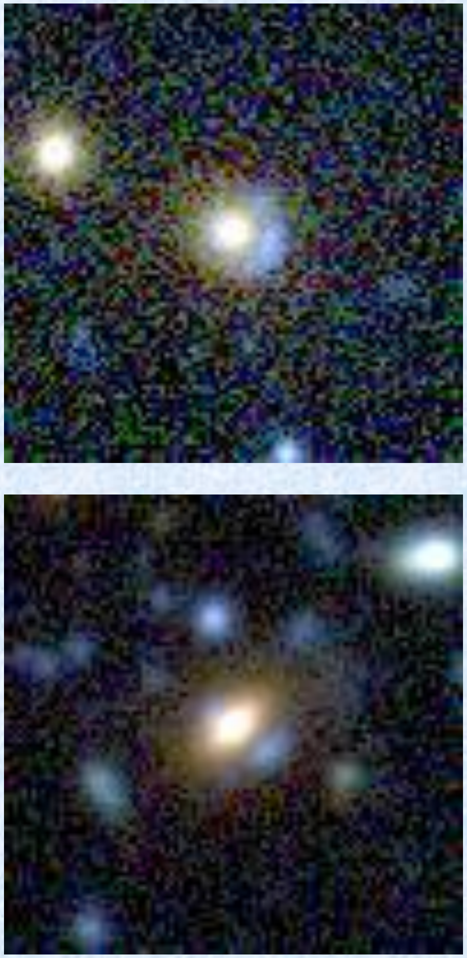
- Half night allocated June 2006:
  - partially cloudy. 1.5 hours in three pointings (effectively 1800s) in Z-band. No detection
- One night allocated September 2006:
  - completely lost to wavefront sensor failure
- With current technology is hard!

Wish list. Improvements:

- Higher redshift targets. At Z, OSIRIS field of view is too small, need mosaicing with loss of time:
  - Much better at longer wavelengths. 3.2”x6.4” or 4.8”x6.4” is larger than lens (typically <3”). Dither on targets?
- Brighter targets:
  - With full SLACS (88 vs 23 last year) or SL2S we can find even brighter emission lines
- Higher Strehl ratios:
  - reduce exposure times and thus make it practical to collect sizeable samples. **(NGAO!)**

SL2S

- Ground based selected candidates (from CFHT-LS)
- AO-NIRC2 to confirm and exploit scientifically (hopefully in 2007B)
- Lens redshift z~0.7
- Source redshift z~1.4 (Ha in H band!)



Summary

- AO + integral field spectroscopy + kinematic lensing =
  - Virgo-like resolution at cosmological distances
  - Velocity fields/masses (Tully Fisher..)
  - Improved mass models
    - Source/lens decomposition in emission line image
- Currently hard with SLACS sample and present capabilities/time allocations. Things may improve with SL2S.
- **With NGAO this should work very well!!**

The end

Constructing χ^2

Need to compare:

observed image plane with *model source plane*

Recall lens equation:

$$\beta = \theta - \alpha(\theta; m)$$

"Forward":
Given true position, βθββ, find observed position(s), θ.
*Difficult!

"Backward":
Given observed position, θ, find true position, β.
*Easy!

... but it is not straightforward to map pixellated data onto source plane.

Creating 'Model' Image Plane

Convolve surface brightness with instrumental PSF.

Using exposure time, zero point, convert SB to counts.

$$\boxed{N^{model}\left(\vec{r}_{i}\,;\,\vec{\boldsymbol{m}}\,,\,\vec{\boldsymbol{s}}\right)}$$

Perform weighted convolution for line of sight velocity.

Ignore points for which no velocity measurement is possible.

$$\boxed{v_{z}^{model}\left(\vec{r}_{i}\,;\,\vec{\boldsymbol{m}}\,,\,\vec{\boldsymbol{u}}\right)}$$

Finding Best Parameters

Finally, compute Ξ^2 for given parameters:

$$\chi^2(m, \vec{s}, \vec{u}) = \sum_i \left(\frac{\left(N^{obs}(\theta_i) - N^{model}(\theta_i; \vec{m}, \vec{s})\right)^2}{\sigma_N^2(\theta_i)} + \frac{\left(v_z^{obs}(\theta_i) - v_z^{model}(\theta_i; \vec{m}, \vec{u})\right)^2}{\sigma_{v_z}^2(\theta_i)} \right)$$

Minimize $\chi 2$ over all free parameters.

This can be challenging since minimizing over many parameters.
(Recall, simple example has 13 parameters!)

Broyden-Fletcher-Goldfarb-Shanno method is efficient:
Gives set of best parameters: $\{\hat{m}, \hat{s}, \hat{\boldsymbol{u}}\}$

Also gives *approximate covariance matrix* at minimum.

MCMC Basics

Markov Chain Monte Carlo (MCMC) allows us to sample points from an arbitrary probability distribution, P.

Given a probability distribution P(a) that we can evaluate for any a, create a 'chain' of points using the following rules:

1. From a_i draw a new poisition a' from a *proposal distribution, Q(a',a)*.

2. If $P' > P_i$: $a_{i+1}=a'$
   If $P' < P_i$: $a_{i+1}=a'$ with probability P'/P_i, otherwise $a_{i+1}=a_i$.

Results in $\{a_i\}$ sampled from true probability distribution, P.
Independent of starting point, a_0, and proposal distribution, Q.

Need multiple chains to test for convergence:
variation within each chain = variation between chains

Efficient MCMC

MCMC is useful when dealing with parameter spaces with many dimensions.

Likely that some parameters are degenerate.
(e.g. V_{max} and r_0)

Sampling with fixed step sizes along parameter axes is inefficient.

Instead, can use steps given by diagonalizing covariance matrix.