Measuring AO Performance

Julian C. Christou and Donald Gavel

UCO/Lick Observatory

CfAO 2006

Adaptive Optics Performance

How to measure it from focal plane images?

- **Conventional approach is using the Strehl Ratio.**

$$S = \frac{h(r)_{\mathrm{pk}}}{p(r)_{\mathrm{pk}}} = \frac{h(0)}{p(0)} \qquad \text{where} \quad 0 \le S \le 1$$

where both are normalised to the same volume

- **Exactly how best to measure Strehl is currently being investigated.**

This depends upon generating the perfect PSF; the presence of additive noise (detector and photon); image plane sampling; the effects of incorrect bias subtraction and flat-fielding, finding the actual peak-location etc.

Measuring Image Quality

- Other Approaches besides Strehl Ratio
- Image Sharpness (originally described by Muller and Buffington, 1974)

S_1 - Size of PSF

$$S_1 = \frac{\sum h_i^{\,2}}{\left[\sum h_i\right]^2}$$

Advantage – independent of knowing peak location and value. - Can be applied to extended sources.

Disadvantage – The numerator is contaminated by an additive noise term $\approx n^2$.

S_3 - Normalised peak value – directly related to Strehl Ratio

$$S_3 = \frac{h_{\mathrm{pk}}}{\sum h_i}$$

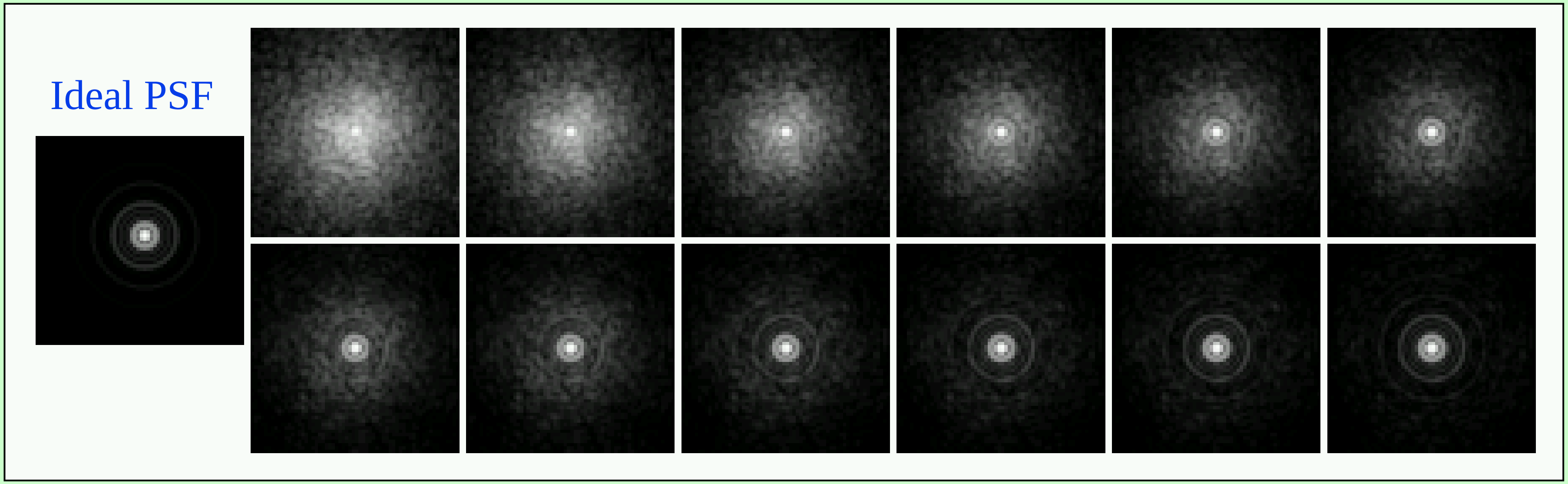
$$SR = \frac{h_{\mathrm{pk}}}{\sum h_i} \Big/ \frac{p_{\mathrm{pk}}}{\sum p_i} = \frac{S_3(h)}{S_3(p)}$$

Disadvantage – sensitive to measurement of peak location and value.

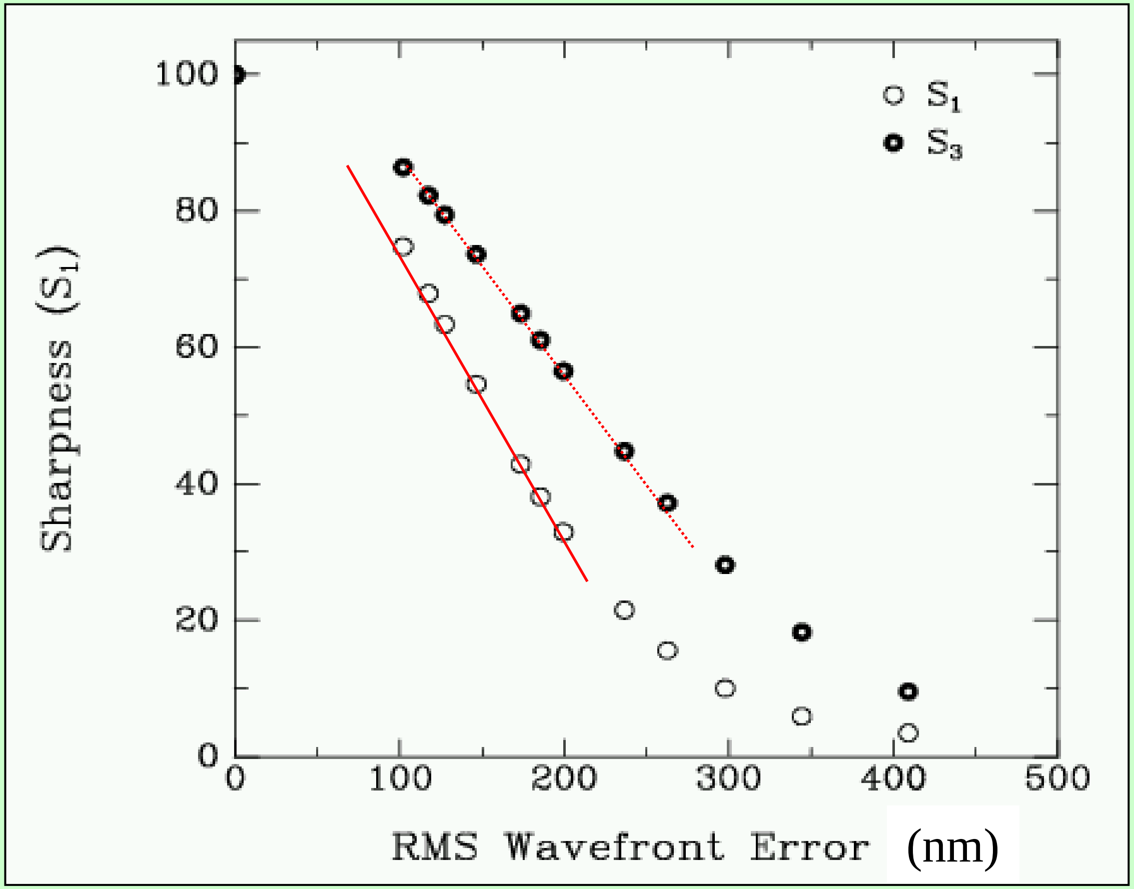
Advantage – No noise bias

Synthetic Data

1. Palomar pupil geometry: primary mirror diameter of 4.88m and a central obscuration of 1.8m. No secondary supports modelled.
2. H-band (1.65 microns) with different levels of AO correction.



Adaptive Optics Performance - Sharpness

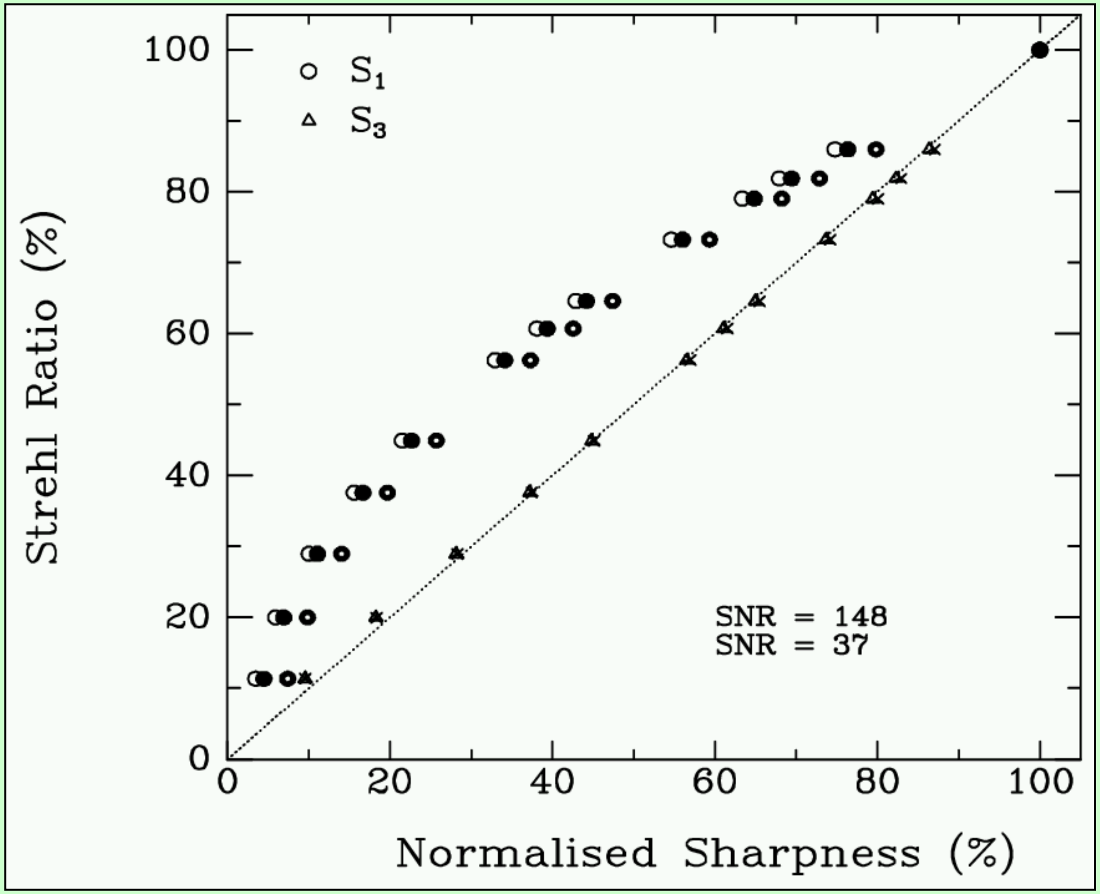


- Sharpness criteria compared with residual wavefront error from the simulations.

- S_1 has a steeper slope for smaller rms phases.

S_1 – -0.45 nm^{-1}
S_3 – -0.30 nm^{-1}

Adaptive Optics Performance - Sharpness



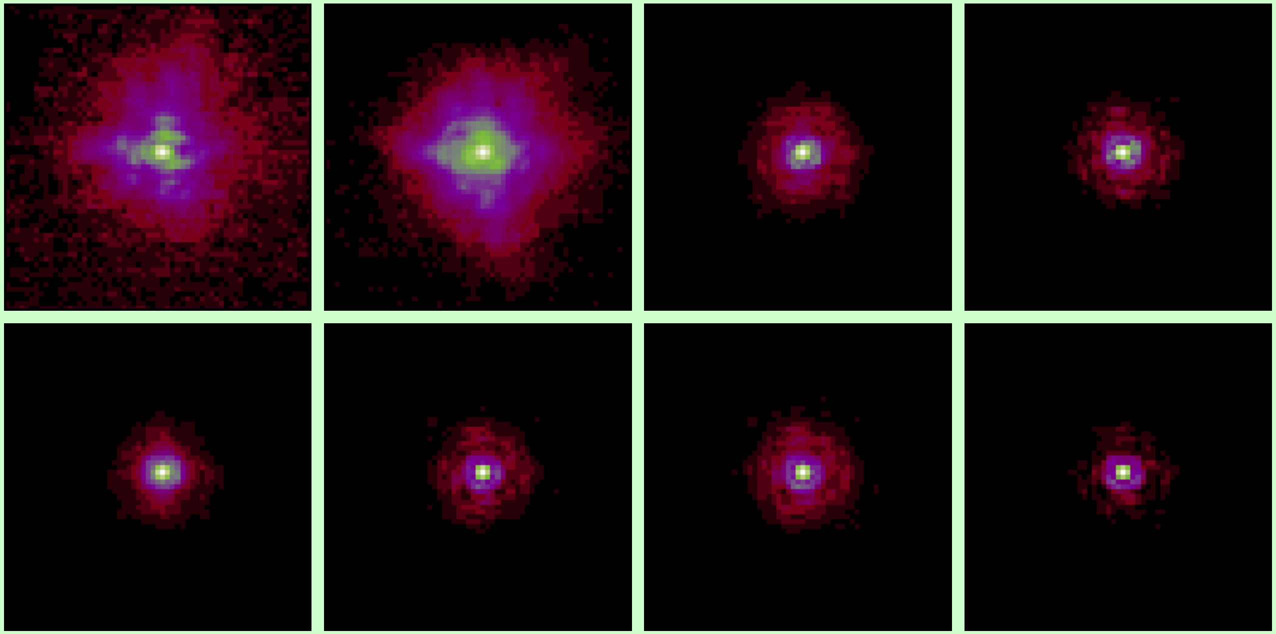
Relationship between S_1, S_3 and the Strehl Ratio.

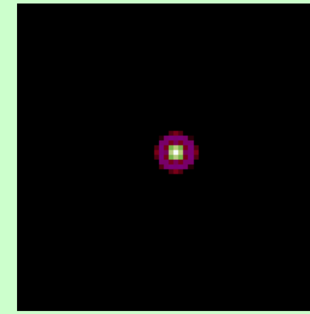
S_1 and S_3 values generated from noise-free simulations as part of the CfAO Strehl study.

Both S_1 and S_3 are normalised to those of the ideal PSF.

The effect of constant noise is shown on S_1.

Measured Point Spread Function

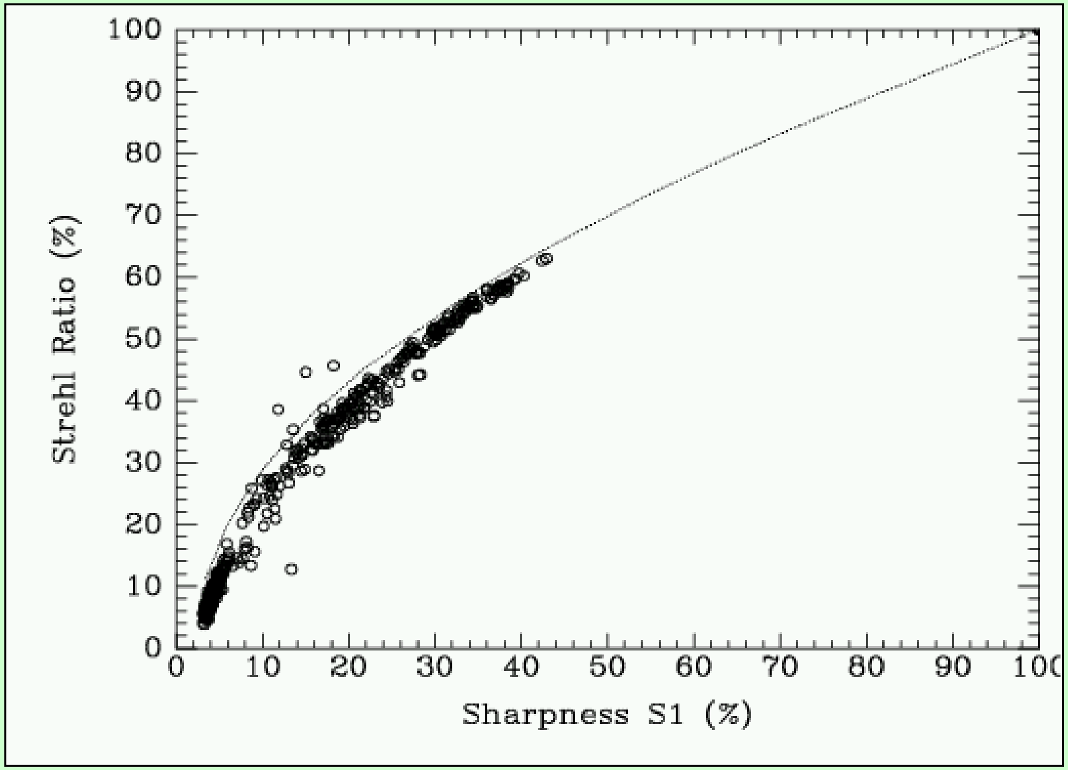




Ideal PSF

Variation in NGS PSF quality from the Lick AO system (all at 2 microns)

Adaptive Optics Performance - Sharpness



- Sharpness (normalised S_1) compared with Strehl ratio for NGS Lick AO data.
- Data obtained with different SNR, observing conditions, nights.
- Dashed line obtained hueristically from the noiseless simulations.

Departure from simulations could be due to either overestimating S_1 (e.g. presence of noise) or underestimating Strehl ratio (not accurately locating the peak). Further analysis on noisy simulations needed.

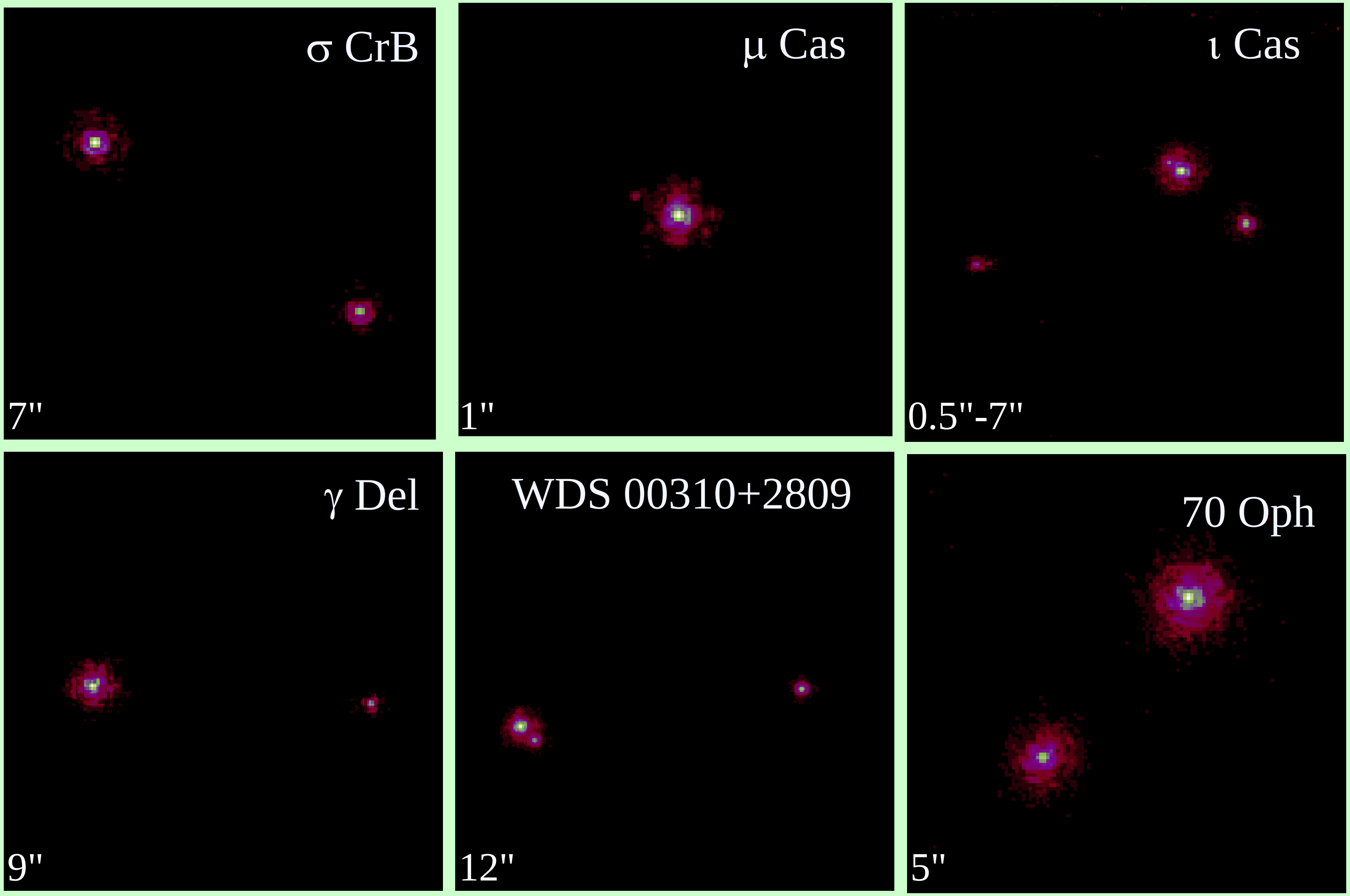
Accuracy of system performance measurements can be obtained from *SR* and S_1.

Binary Star Measurements

- Science Targets
  - Basic Astronomy; stellar classification; stellar motion – orbits
- AO Performance
  - Isoplanatic Issues – on-axis vs. off-axis performance
  - Isoplanatic angle - θ_o
- Analysis Performance
  - Measurement of Photometry and Astrometry
- Lick Observatory Data
  - NGS
  - 0.5" ≤ Separations ≤ 12"

Binary Star Measurements

Lick NGS Data



Anisoplanatism via Strehl Ratio

- Binary stars permit direct measurement of anisoplanatism by comparing the PSFs.

- An effective measure of anisoplanatism is the fall off of the Strehl ratio of the off-axis source compared to the on-axis source.

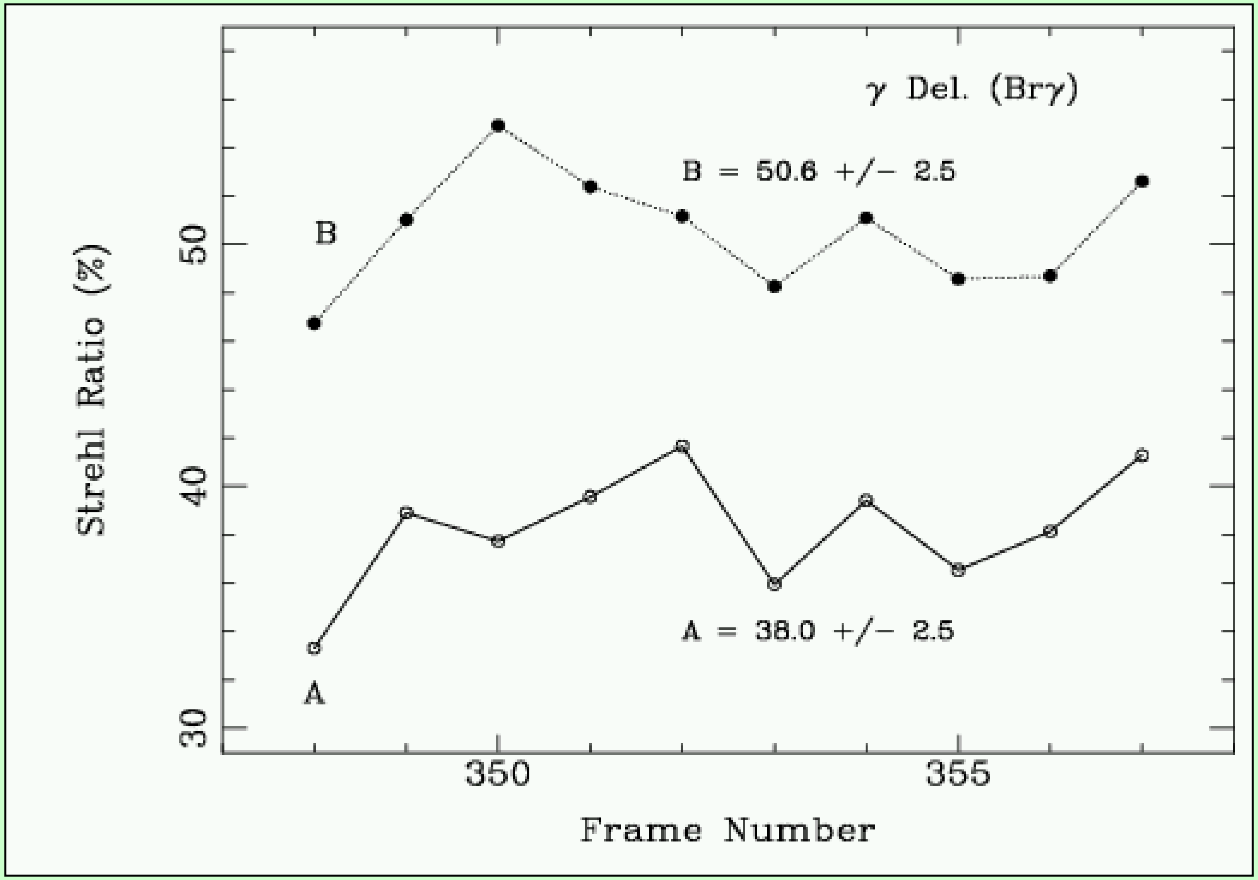
$$\frac{SR_{\text{off-axis}}}{SR_{\text{on-axis}}} \approx \exp\left[-\frac{\theta}{\theta_{\text{o}}}\right]$$

where θ is the binary separation

Anisoplanatism via Strehl Ratio

- γ Del (sep = 9.22 arcseconds) – ratio = 0.76 ± 0.04

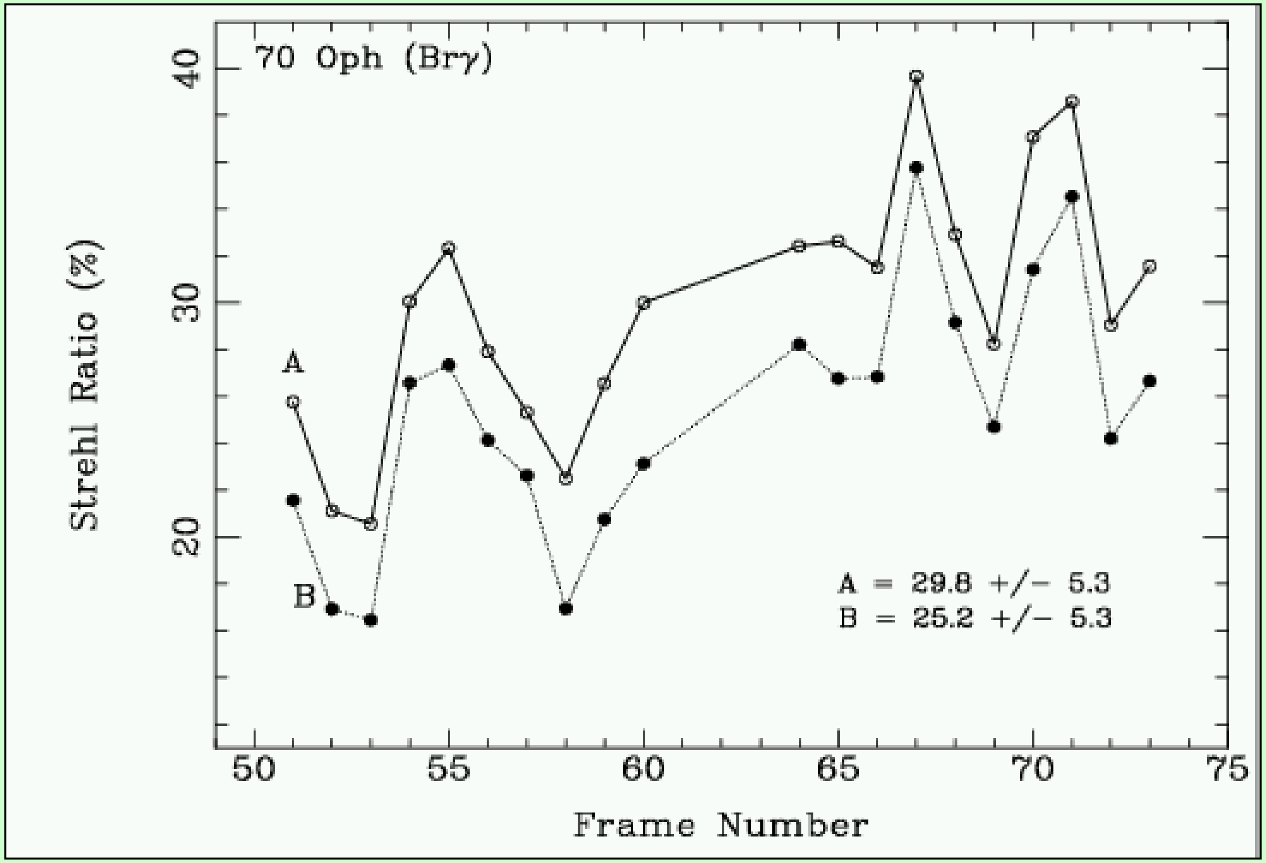
θ_o= 20.1" ± 2.1"



Anisoplanatism via Strehl Ratio

- 70 Oph (sep = 4.79 arcseconds) – ratio = 0.84 ± 0.04

θ_o = 14.3" ± 2.5"



Anisoplanatism via Strehl Ratio

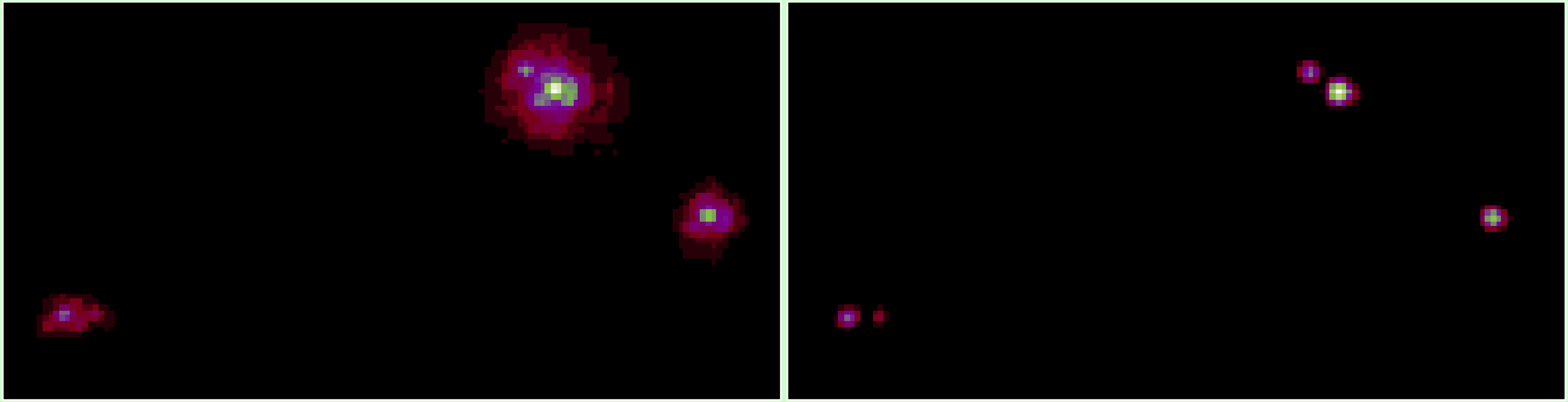
Summary of Binary Strehl Ratio Measurements

- Strehl ratio changes vary similarly for both components.
- Strehl ratio is quite variable for a set of observations (≈ seconds - minutes) up to changes of 20%.
- Differential Strehl ratio also varies – relative position on the detector?
- Isoplanatic angle (as determined from differential Strehl ratio) also varies with 15" $\leq \theta_o \leq$ 30" with some results implying minutes!

Binary Star Measurements

- **Analysis Techniques**
  - Iterative Blind (myopic) deconvolution (Christou-CfAO)
  - Parametric Blind Deconvolution (PSF Modelling) (Drummond-AFRL)

- **Astrometry and Photometry**
  (on following pages)



Binary Star Measurements

Summary of Astrometry and Photometry

- Astrometry between the two techniques shows good agreement (≈ 0.001")
- Differential Photometry is in general good agreement (≈ 0.02 mag) with a few exceptions.

- σ CrB ($\Delta J = 0.5$)
- μ Cas ($\Delta J = 0.4$; $\Delta Br\gamma = 0.2$)
- ι Cas Aa ($\Delta J = 0.2$; $\Delta K_s = 0.2$)
- ι Cas Ac ($\Delta H = 0.15$)

Christou, J.C., Drummond, J.D., *Measurements of Binary Stars, Including Two New Discoveries, with the Lick Observatory Adaptive Optics System*, The Astronomical Journal, Volume 131, Issue 6, pp. 3100-3108.

Astronomical AO System Data Analysis

Julian Christou (UCSC)

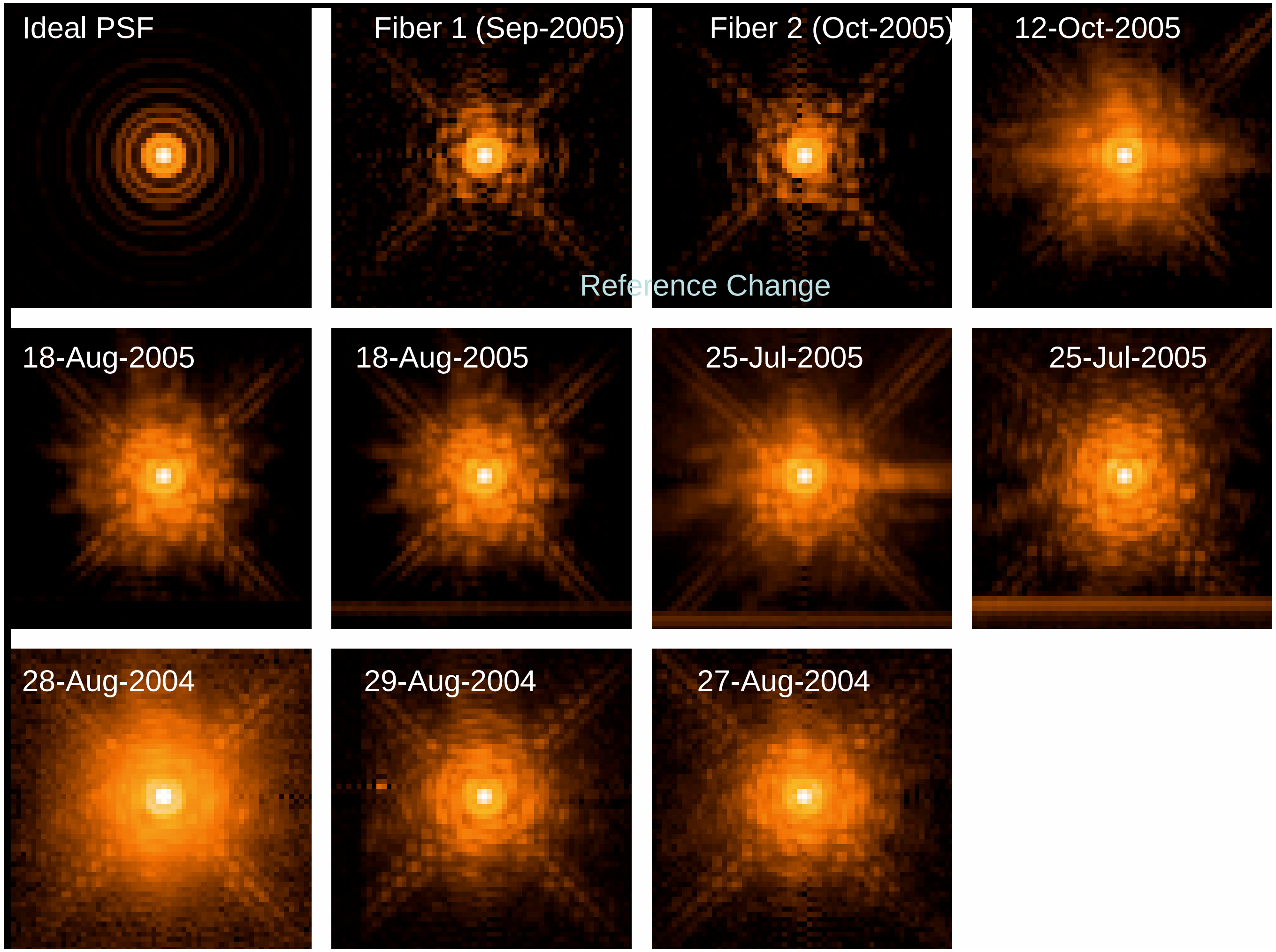
Szymon Gladysz (NUI)

Gladysz, S., Christou, J., Redfern, M., *Characterization of the Lick adaptive optics point spread function*, SPIE Proc., 6272, June, 2006

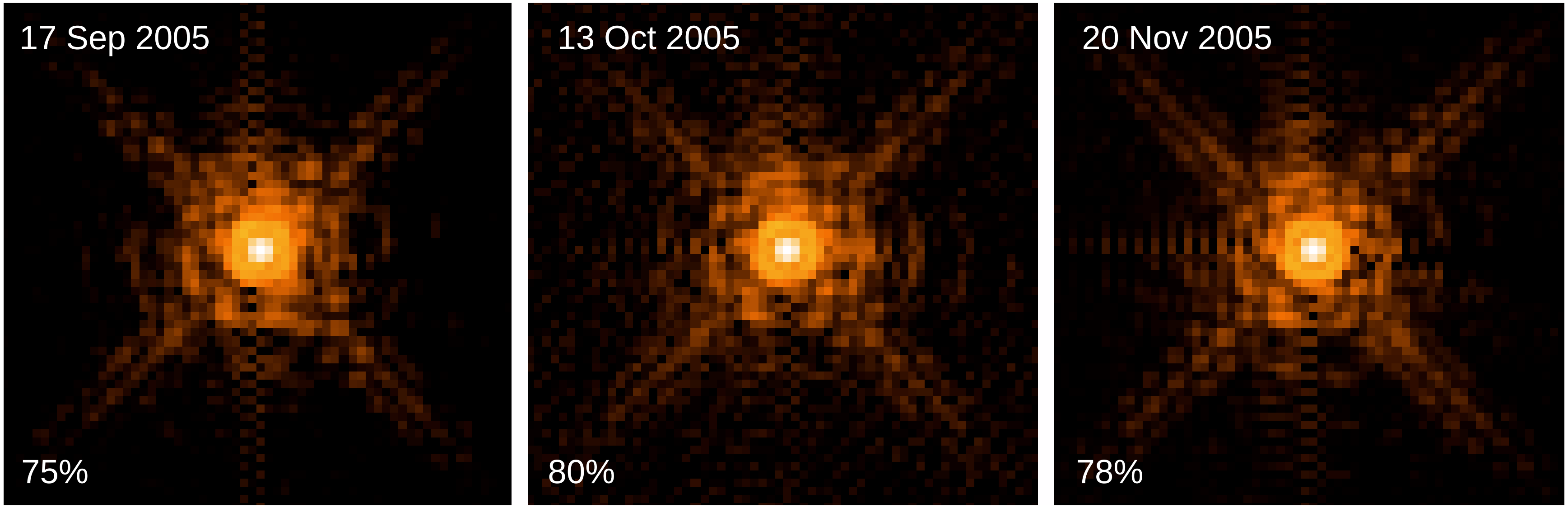
High Speed PSF Measurements

- Data sets obtained at Lick almost monthly between July 2005 and Feb 2006.
- IRCAL *fastsub* mode ("freeze" images)
  - t_{exp} = 22*ms* and 57*ms*
  - Duty cycle ~ t_{exp} + 30*ms*
- field size of 4.864 × 4.864 arcseconds (64 × 64 pixels)
- Target objects: m_v ~ 6-8
- Typically 10 sets of data each of 1000 frames - 10^4 total frames

Long Term PSF Stability



Lick AO Fiber Source

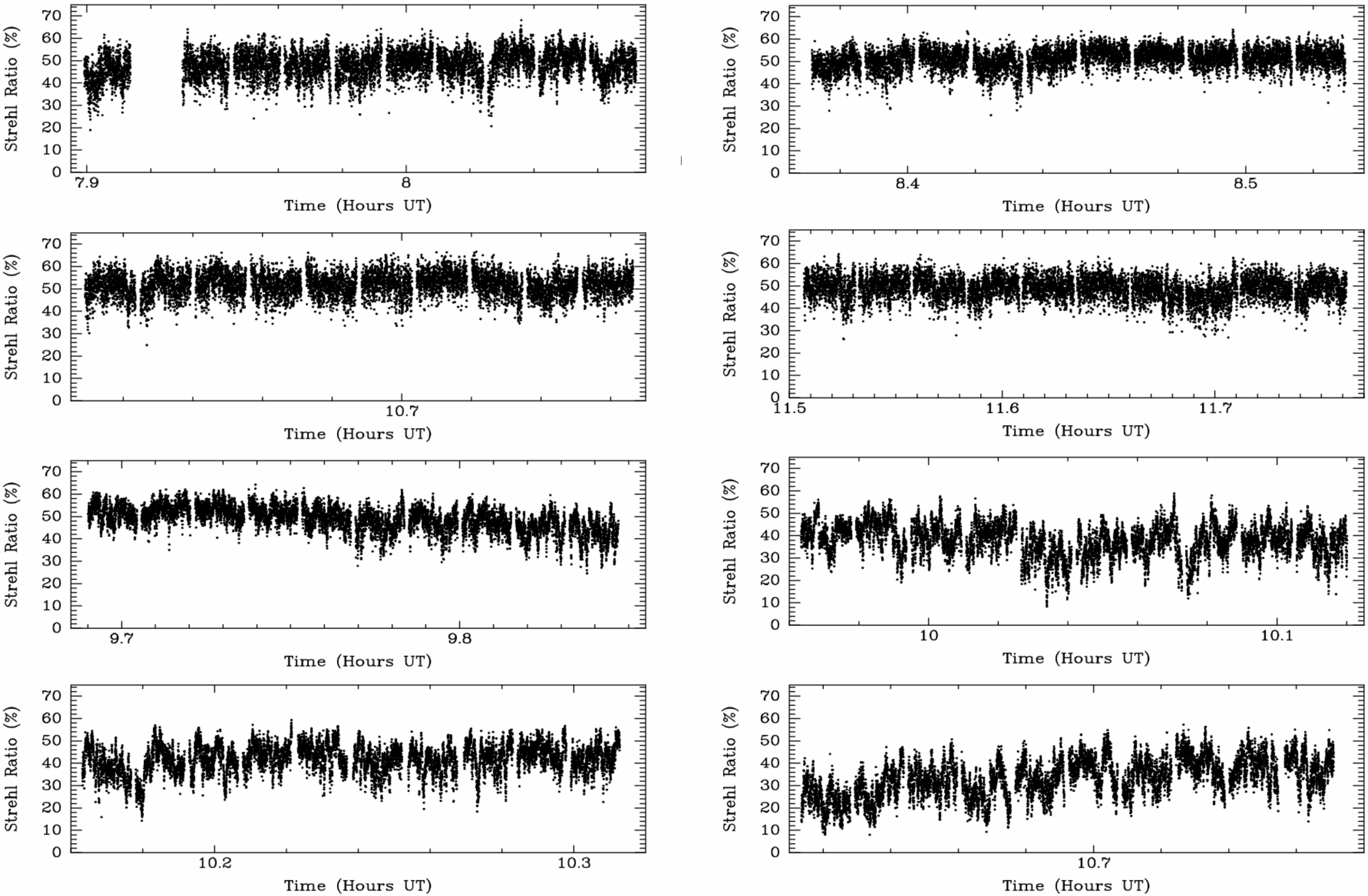


- Stable structure in atmospheric-free PSF
- Strehl Ratios typically 75% -- 82%

PSF Structure

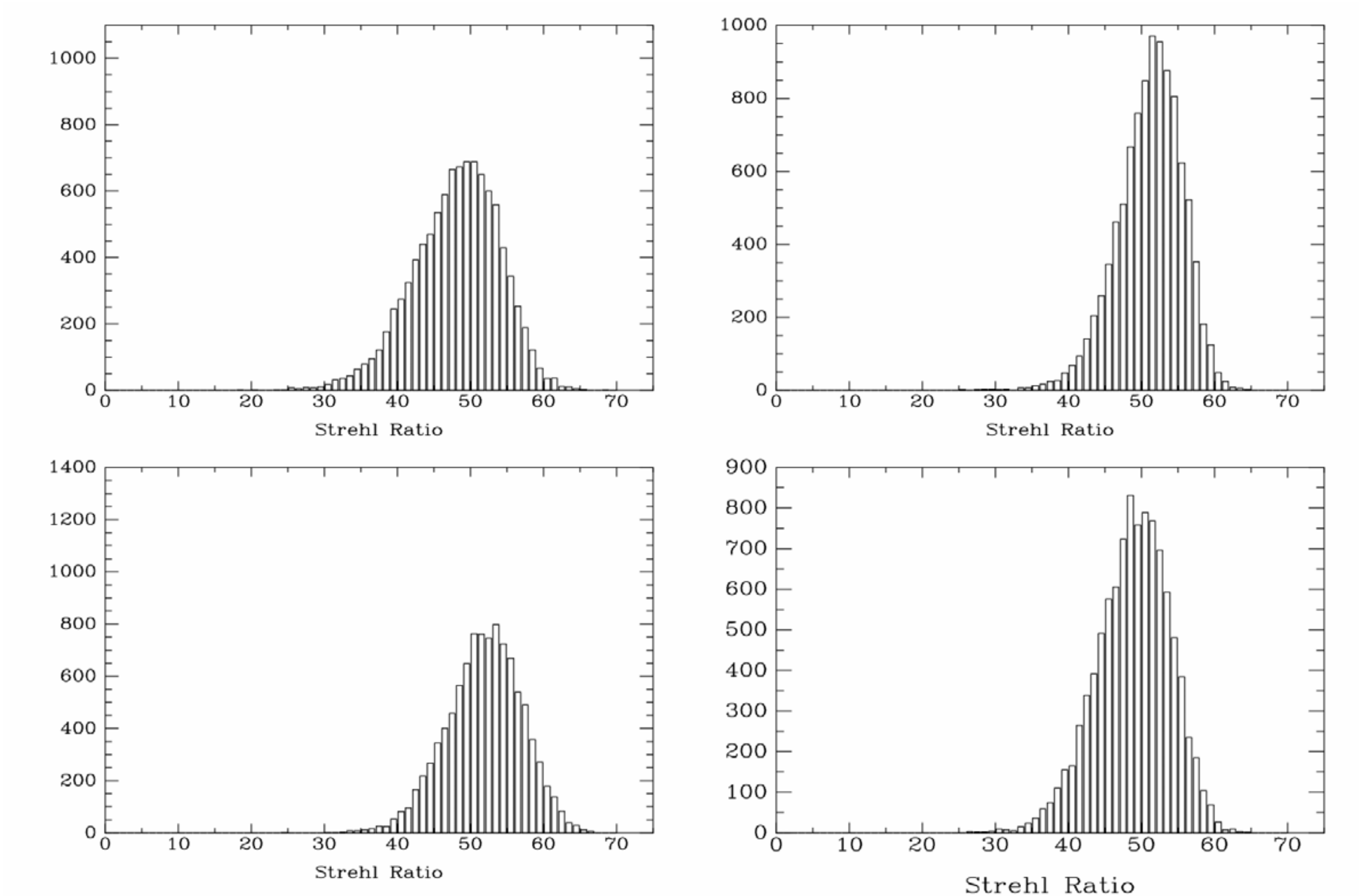
- Fiber Source no better than ~ 80% Strehl ratio.
  - What's the best we can do - 90-95%?
- Strong high-order Residual Aberration limiting performance.
  - Relatively stable over minutes → hours → days → months → years!
  - No significant change with change of DM references
  - Where is this from?
    - DM flatness
    - Unsensed aberrations in main path
    - Non-common path errors
    - Incorrect SH References
    - Obtain Wavefront map from Phase Retrieval/Diversity measurements.
  - Typically the image is "sharpened" on the sky
    - Relative peak value metric - other metrics e.g. S_1
    - First 10 Zernike terms and increasing to 20.
      - Use mirror modes?
- Important to understand for PSF Reconstruction algorithms.
  - We can deal with the atmosphere but can we deal with the system …?

Lick NGS Strehl Stability (10000 frames 22-57ms/frame)



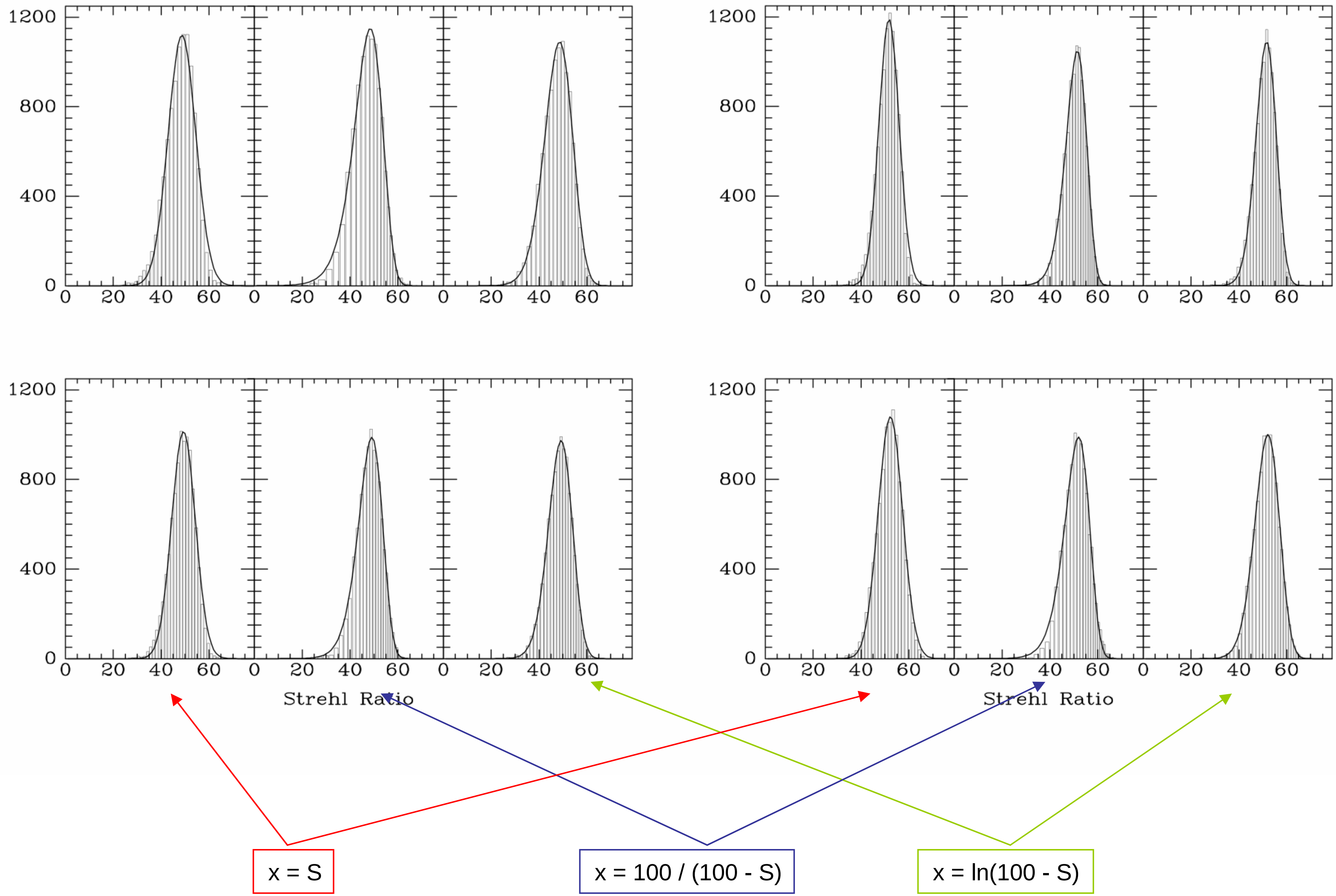
Christou (UCSC), Gladysz (NUI)

Strehl Ratio Distributions

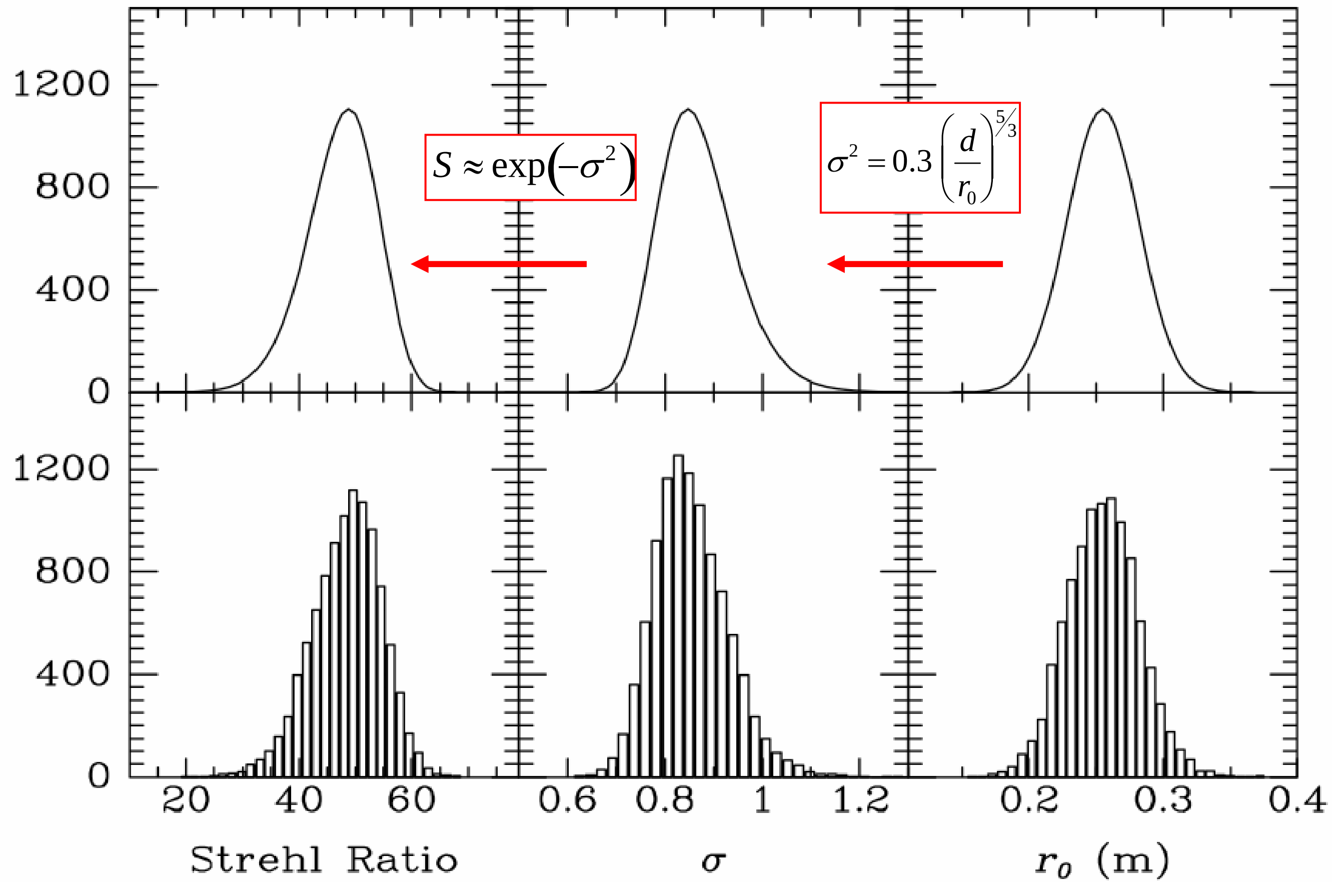


- Distribution of Strehl ratios (for relative stable performance) all show a similar non-gaussian behaviour.
- Similar distributions seen in data from Palomar, Keck and AEOS

PDF Models


1200
800
400
0
0
20
40
60
Strehl Ratio
Strehl Ratio
x = S
x = 100 / (100 - S)
x = ln(100 - S)

PDF Models

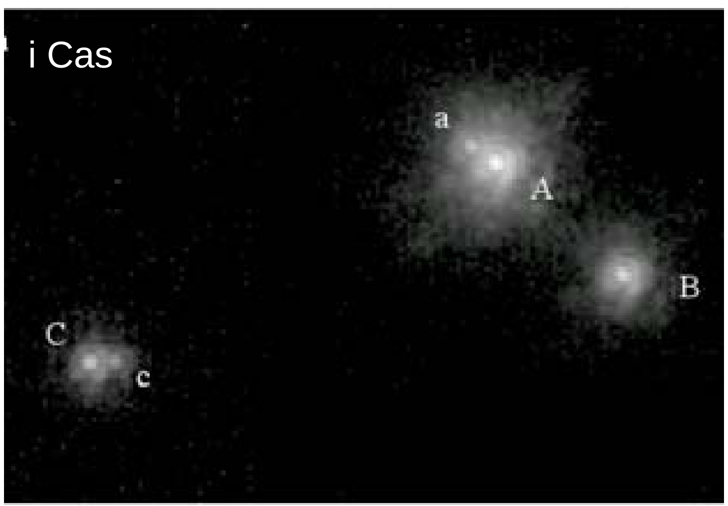


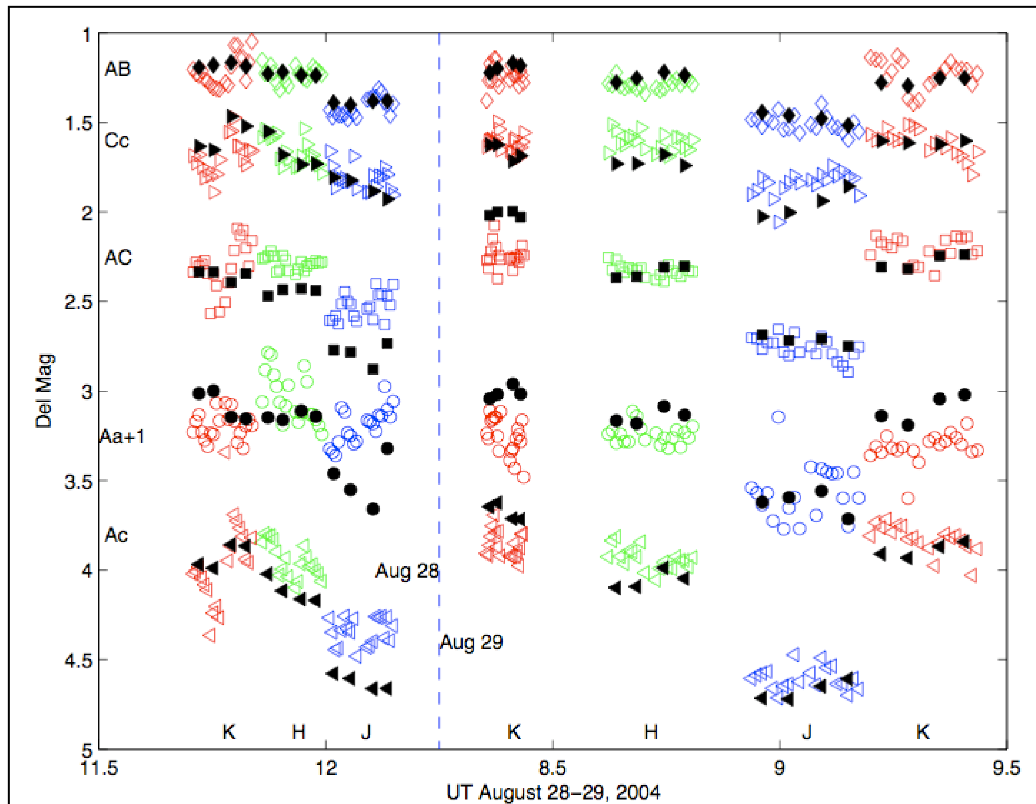
Implication is that the instantaneous Strehl ratio has a Gaussian distribution.

- Using Hudgin and Marachel approximations produces a distribution of Strehl ratios similar to that measured, i.e. skewed to a low Strehl ratio tail.
- Need to obtain simultaneous r_0 and S measurements.
- Speckle noise dominating.

PSF Calibration and Quantitative Analysis

- The complicated nature of the AO PSF makes quantitative analysis problematic.
  - How well does deconvolution preserve astrometry and photometry?
  - Compare with model fitting Techniques

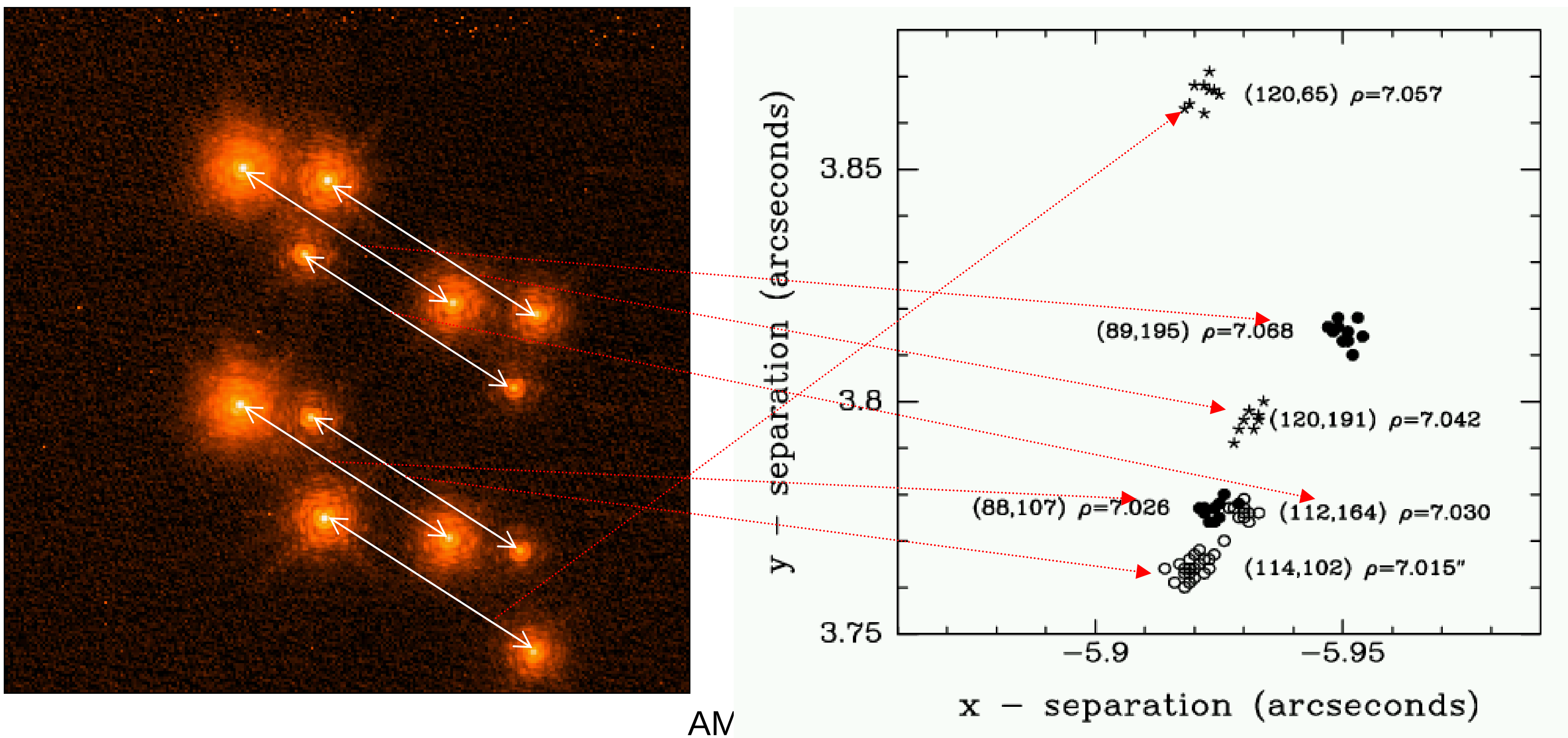




Separation of the components of σ CrB

Sub-pixel peaks located by Fourier interpolation

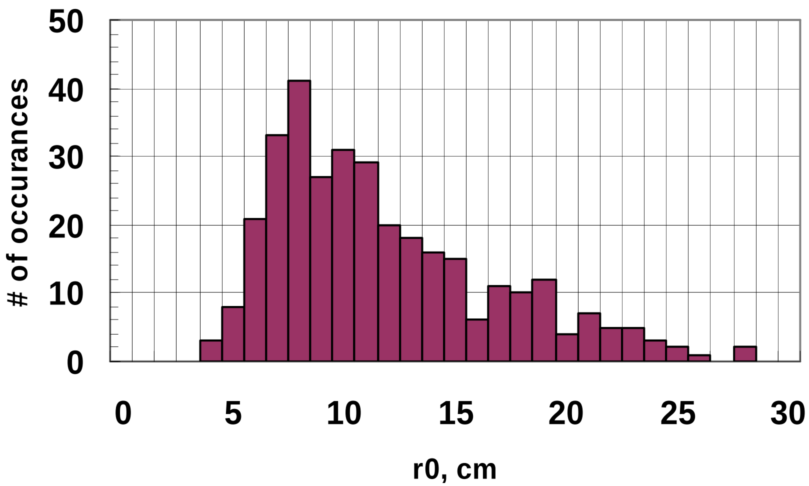
- Six separate measurements of a binary star on different days on different positions on the IRCAL detector.
- Separation depends upon location on detector
- Precision for each location ~ 2 mas (= 0.03 pixels = 1.5% λ/D)
- Separation dispersion ~ 50 mas



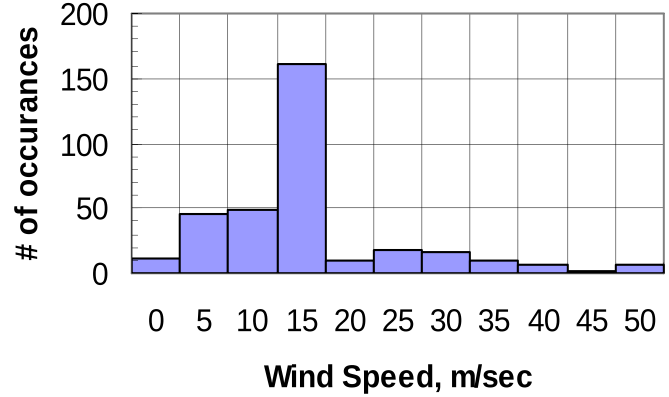
Lick seeing statistics

r0 (Fried seeing parameter) Histogram

median r0 = 10 cm

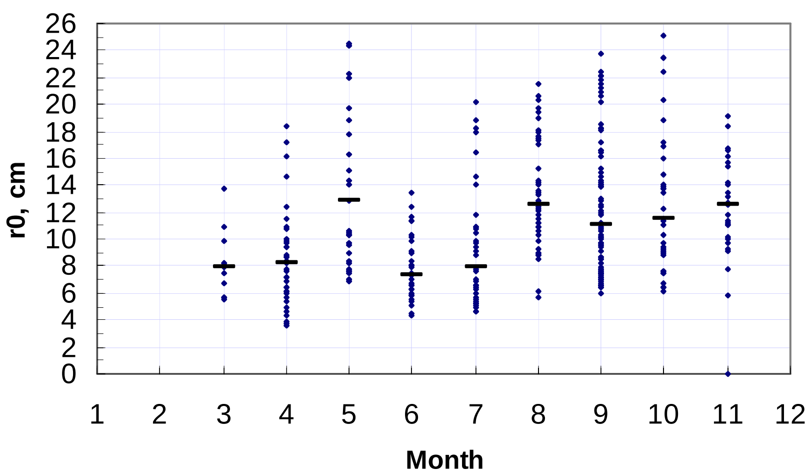


Histogram of Wind Speeds

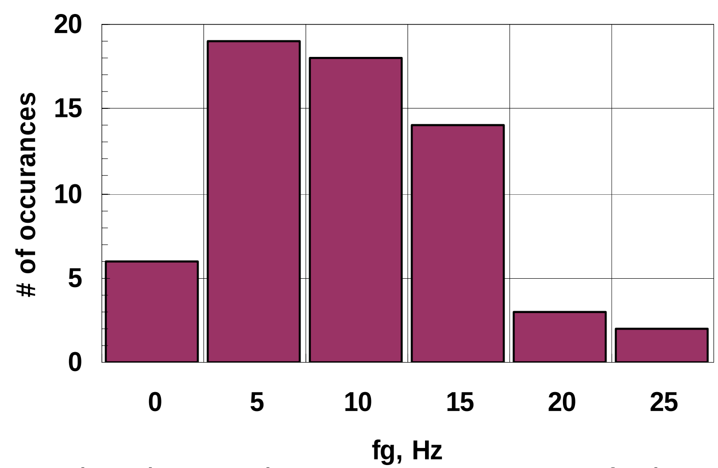


Seasonal Variation of Seeing

(2000-2002)

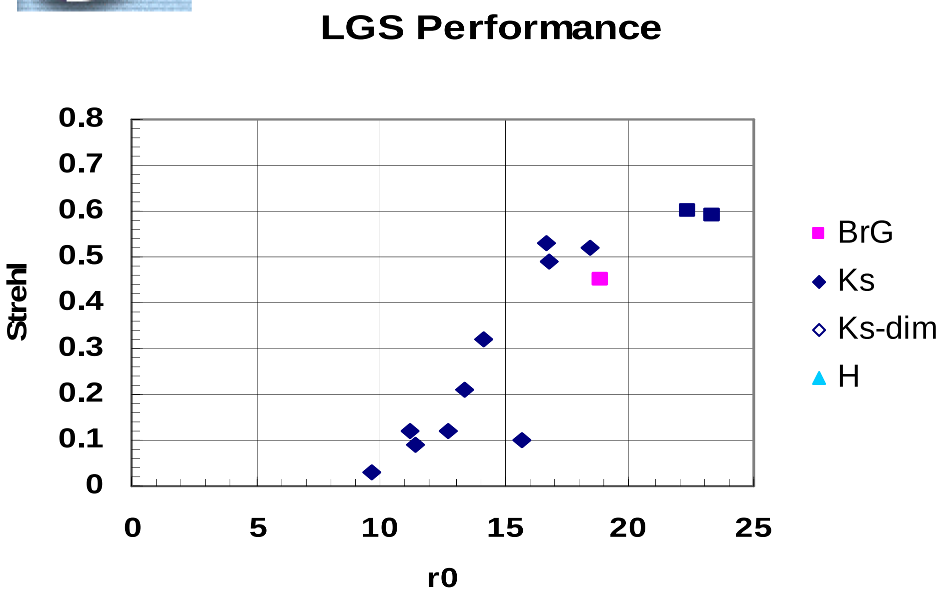


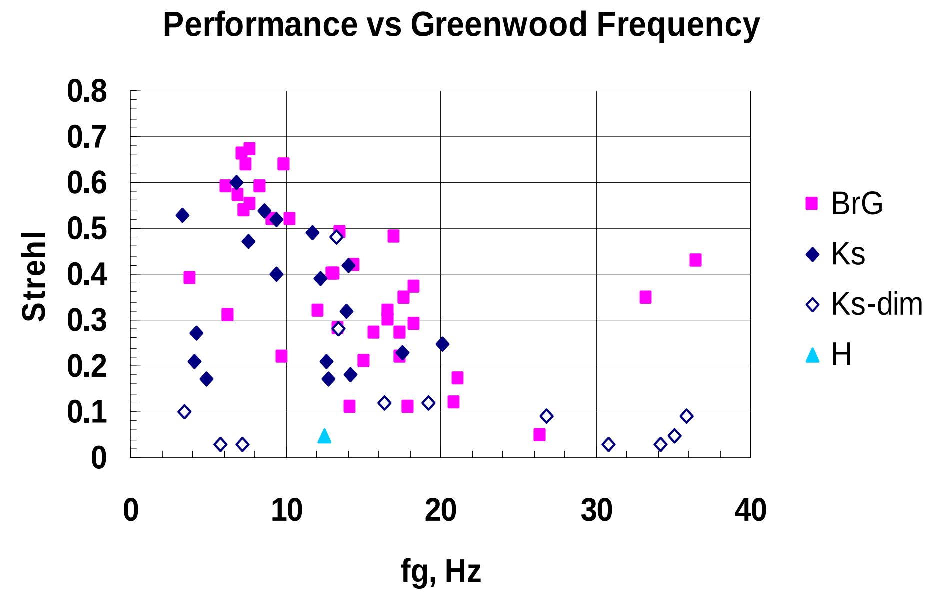
Greenwood Frequency Histogram

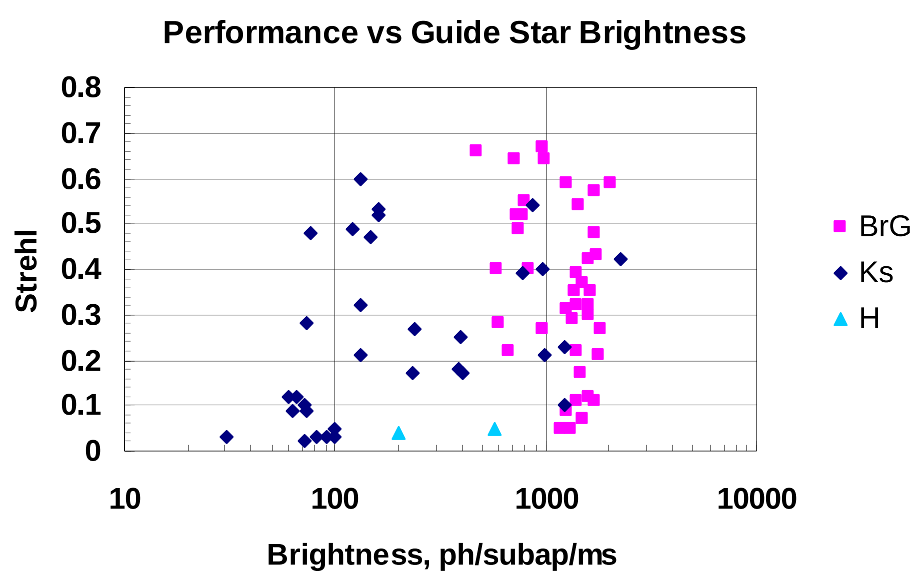


D. Gavel, E. Gates, C. Max, S. Olivier, B. Bauman, D. Pennington, B. Macintosh, J. Patience, C. Brown, P. Danforth, R. Hurd, S. Severson, J. Lloyd, *Recent Science and Engineering Results with the Laser Guidestar Adaptive Optics System at Lick Observatory*, Proc SPIE, 4839, pp. 354-359 (2003).

Lick AO System: performance statistics

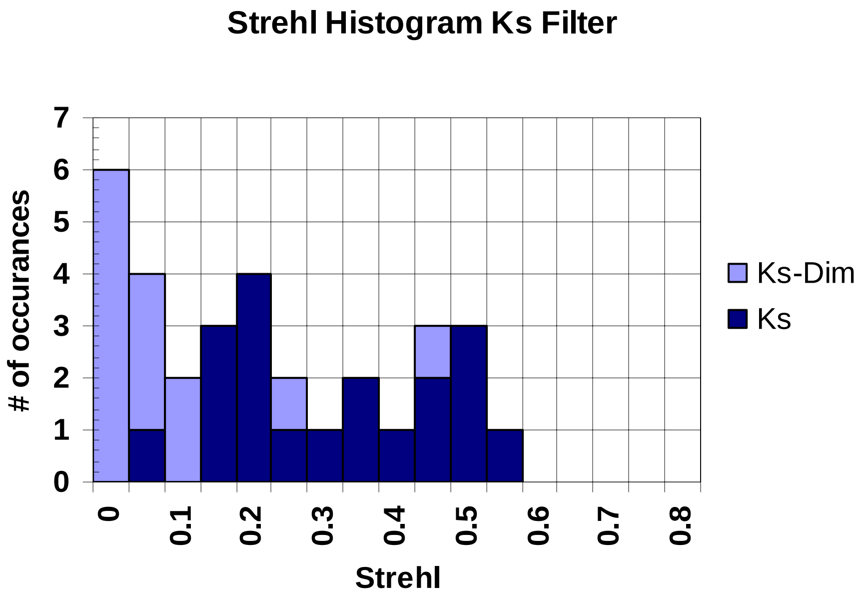


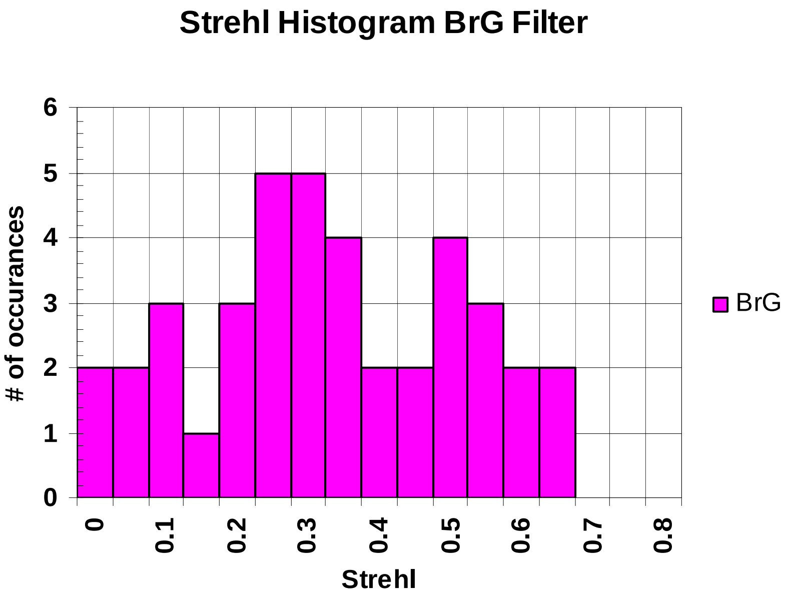




Lick AO System: performance statistics

2001-2002



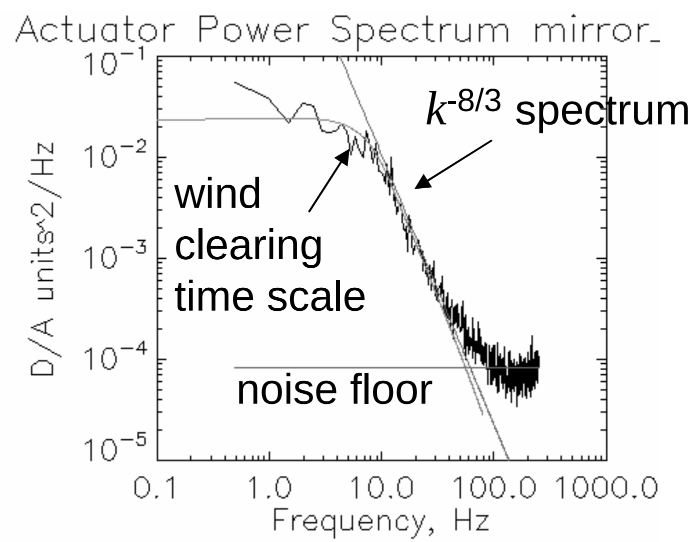


Lick AO System: On-line Performance Analysis

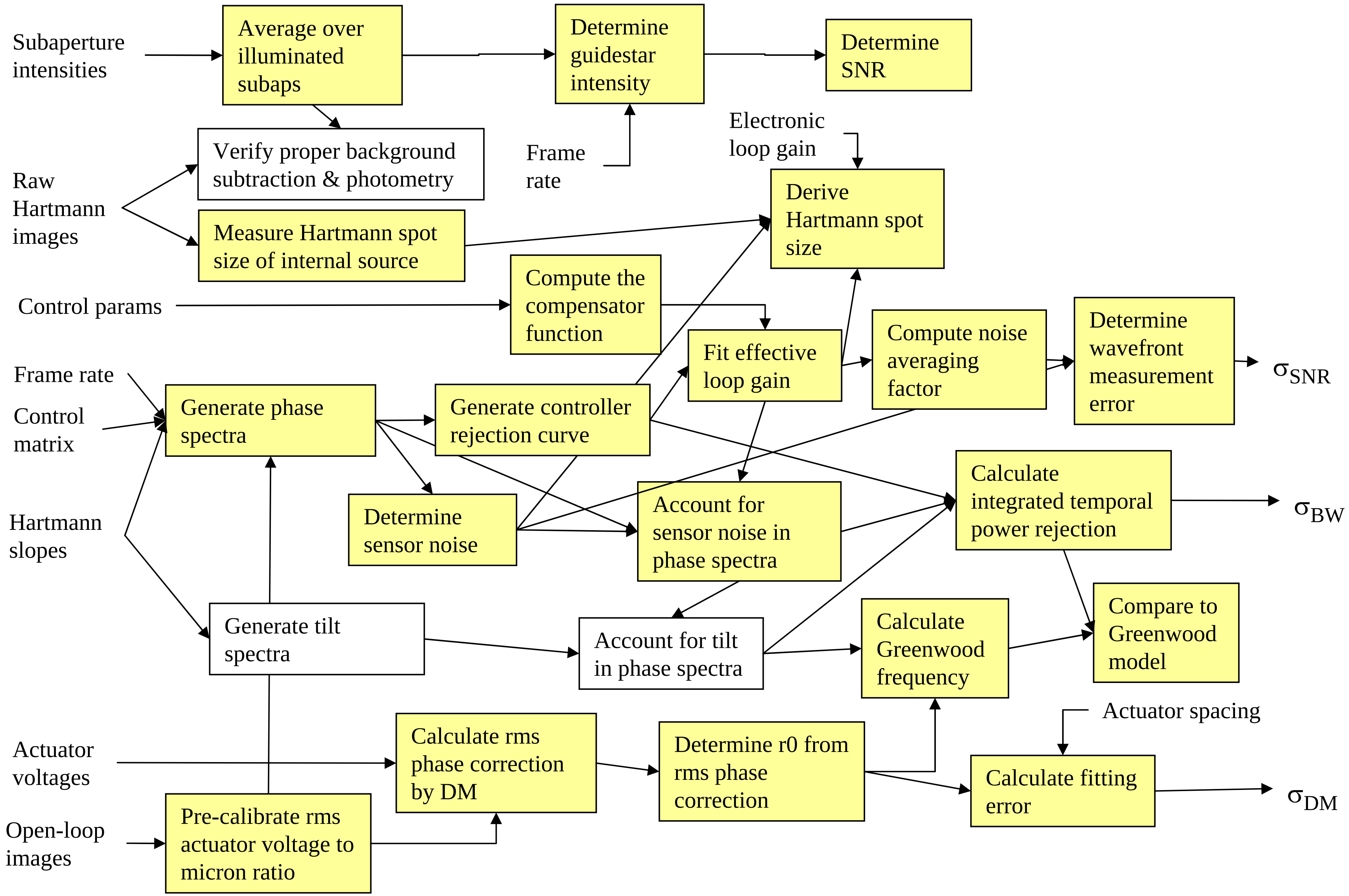
- The spreadsheet errorbudget.xls can help diagnose the sources of Strel loss and aid with on-line AO system parameter adjustments

Fill in the seeing and other system parameters in the green boxes and read the Strehl in the blue box												
					Lick Error Budget					r0	0.15	m
						radians@.55	nm	Strehl@lambdaObs		v-wind	10	m/s
counts	100	photo-electrons			Fitting	2.405180443	210.538	0.70		tau0	0.015	s
read noise	6	electrons			Bandwidth	0.838952777	73.43791	0.96		fg	9	Hz
spot FWHM	2	arcsec			SNR	1.223671374	107.1143	0.91		mu	1	
spot sigma	1.442695	arcsec			Calibration	1.583226455	138.5881	0.85		d	0.43	m
pixel size	2	arcsec			Aniso	0	0	1.00		fs	100	Hz
crosstalk	0.2	arcsec			**Strehl**		283.5481	**0.52**		fc	10	Hz
centroider	quad				FWHM open loop	0.75630429	arcsec at lambdaObs			lambda	550	nm
SNR	6.401844	note: need to load math package			Observed			0.35		lambdaObs	2200	nm
theta	0	arcsec			Unaccounted	2.510864718	219.7891	0.67		calibration	0.85	Strehl (Br
Quad Cell	SNR	4.125565								theta0	6	sec

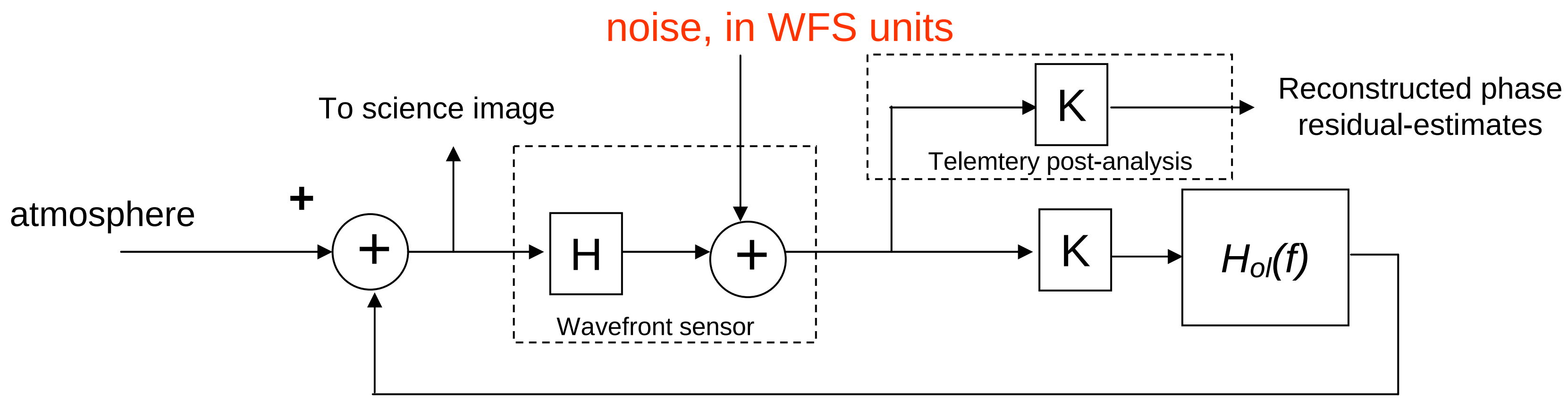
- Other on-line metrics at the operator interface, based on AO system telemetry data analysis:
  - Seeing r_0
  - Wind velocity
  - Temporal power spectrum of turbulence
  - Control loop rejection curves



Lick AO Telemetry Data Analysis Pipeline



Modeling the effect of noise in closed loop



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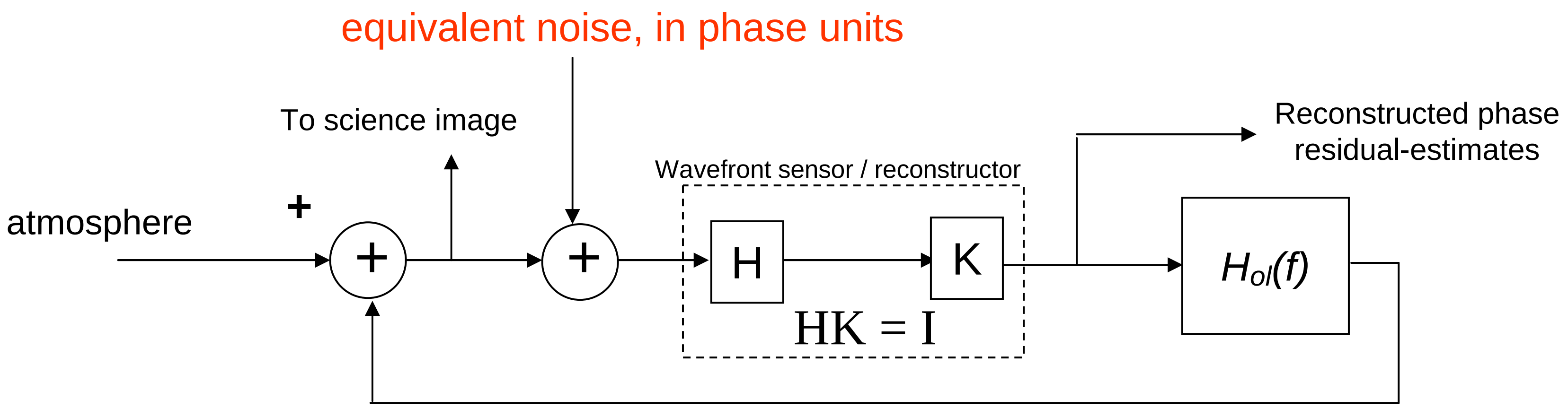
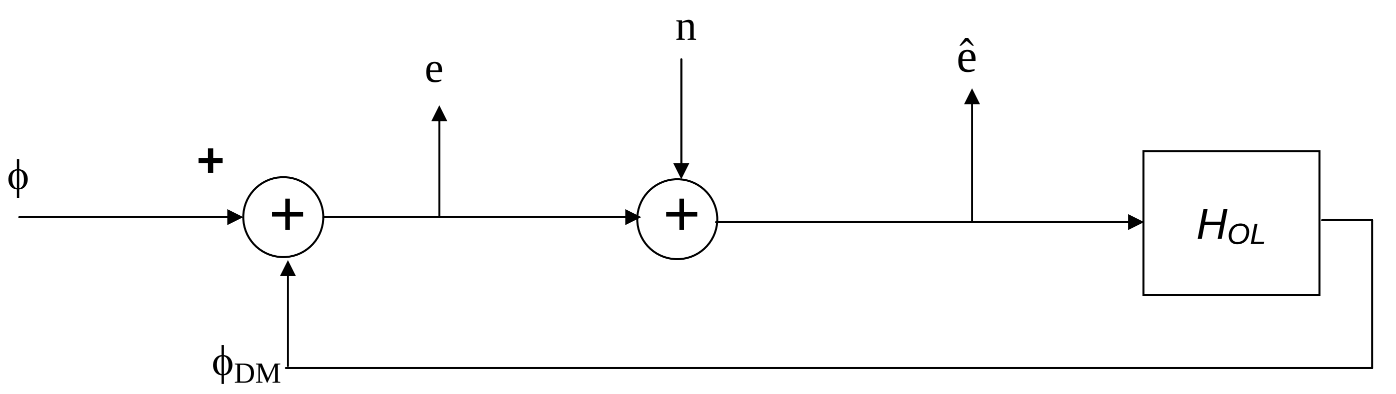


Correcting the closed loop residual phase spectrum for the effects of noise



Closed-loop transfer function: *low-pass*

$$H_{cl}(f) = \frac{H_{ol}(f)}{1 + H_{ol}(f)}$$

"Correction" transfer function: *high-pass*

$$H_{cor}(f) = 1 - H_{cl}(f) = \frac{1}{1 + H_{ol}(f)}$$

$$e = H_{cor}\phi + H_{cl} n$$

$$\hat{e} = H_{cor}(\phi + n)$$

$$\langle \phi n \rangle = 0$$

$$\Rightarrow$$

$$S_e = |H_{cor}|^2 S_\phi + |H_{cl}|^2 S_n$$

$$S_{\hat{e}} = |H_{cl}|^2 S_\phi + |H_{cl}|^2 S_n$$

$$S_e = S_{\hat{e}} - \left[|H_{cor}|^2 - |H_{cl}|^2 \right]$$

==

Lick 3m error budget
/duck5/lickdata/sep00/lgs6data/sep08/cent_07
Saturday 09/09/00 23:03:44 PDT

Fitting Error (sigmaDM)   117.827 nm
d =   42.8571 cm
r0Hv =   13.6763 cm

Servo Error (sigma_BW)   85.8510 nm
fc =   45.9980 Hz
fgHv =   28.5525 Hz
fs =   500 Hz

Measurement Error (sigma2phase)   81.9109 nm
SNR =   45.7691
control loop averaging factor =   0.452526
spotSizeFactor =   0.882759 arcsec

TOTAL: 167.221 nm

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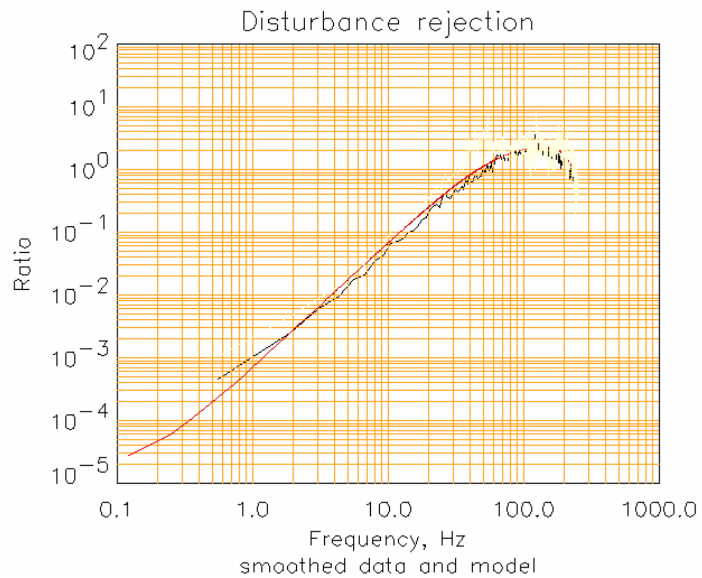

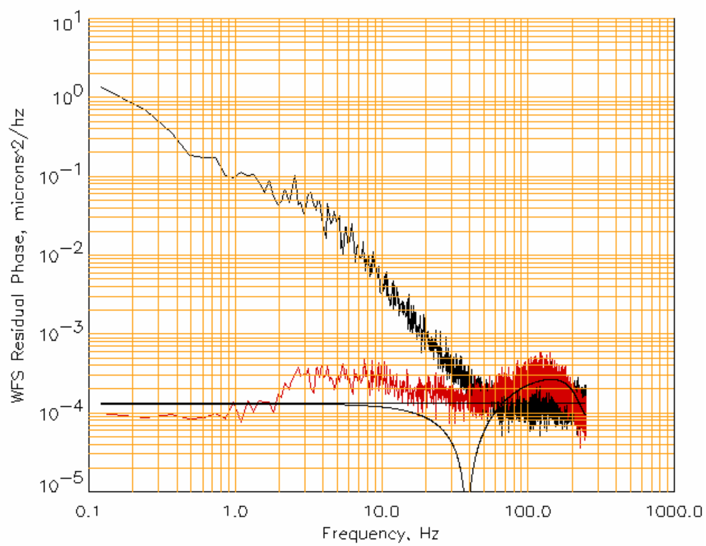

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Lick 3m error budget

/duck5/lickdata/may00/lgs6/may21/cent_03

5/22/00, 5:09 UT

Fitting Error (sigmaDM) 122.912 nm

d = 42.8571 cm

r0Hv = 13.0001 cm

Servo Error (sigma_BW) 174.682 nm

fc = 30.5027 Hz

fgHv = 40.2416 Hz

fs = 500.000 Hz

Measurement Error (sigma2phase) 15.2976 nm

SNR = 100.543

control loop averaging factor = 0.257468

spotSizeFactor = 1.23077 arcsec

TOTAL: 214.138 nm

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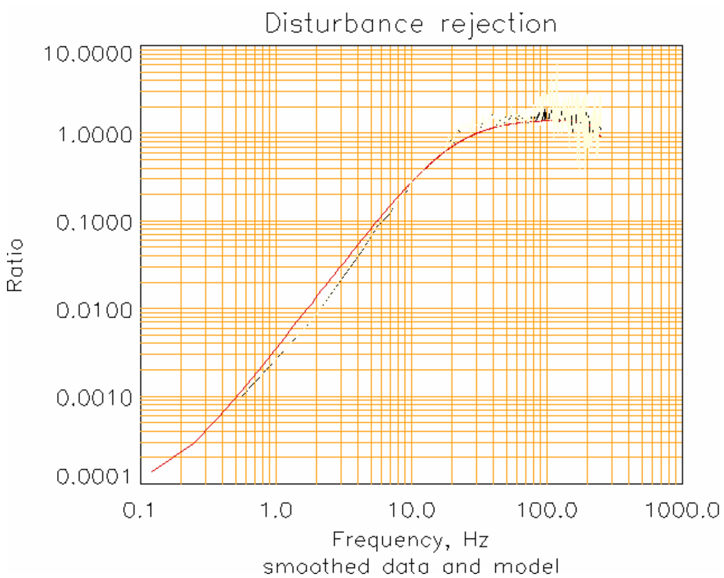
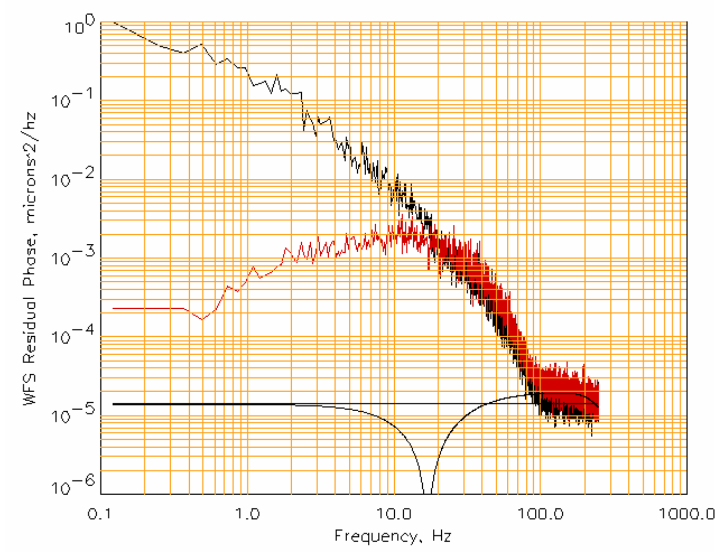




PSFs for Vision Science

Julian Christou

UCSC

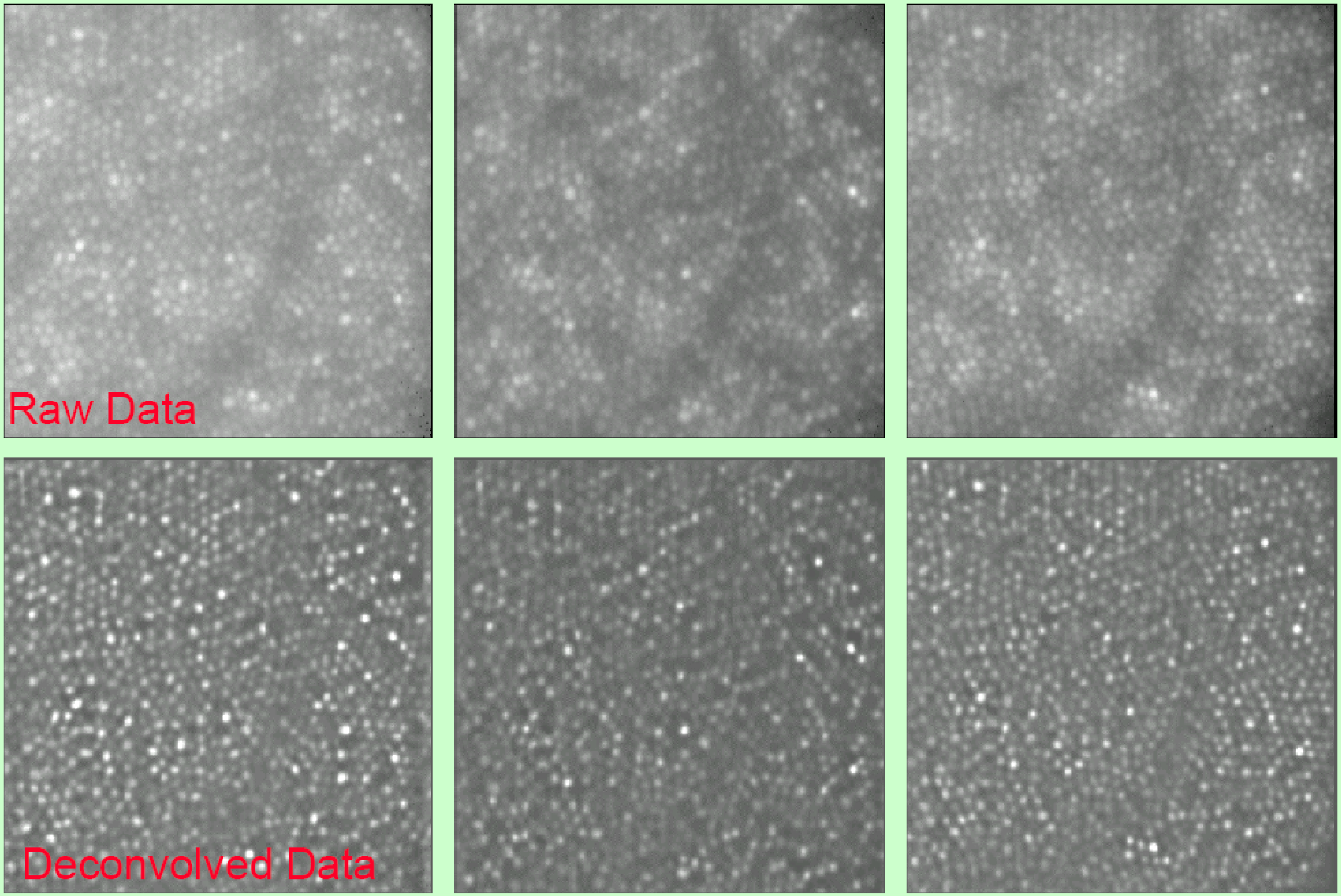
Julian C. Christou, Austin Roorda, and David R. Williams, *Deconvolution of adaptive optics retinal images*, J. Opt. Soc. Am. A 21, 1393-1401 (2004)

Why do we need to know the PSF?

- Knowledge of the PSF is necessary for deconvolution. The more knowledge the better the resulting object information
- Residual uncompensated aberrations in the wavefront leads to increased PSF blurring and structure leading to confusion of object information (especially quantitative).
- A prime example is measurement of cone classification which uses quantitative radiometric measurements from retinal images.

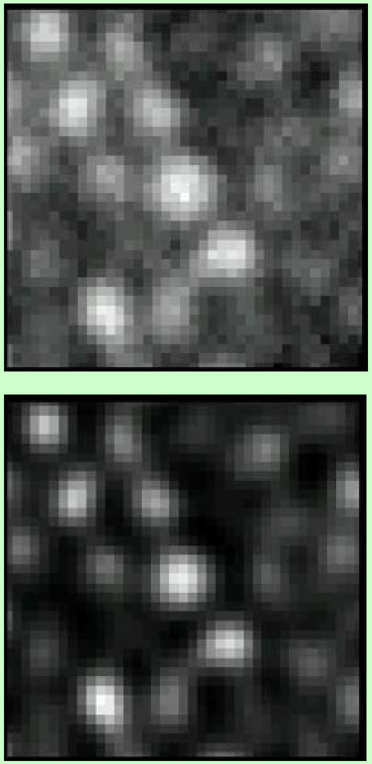
Cone Classification

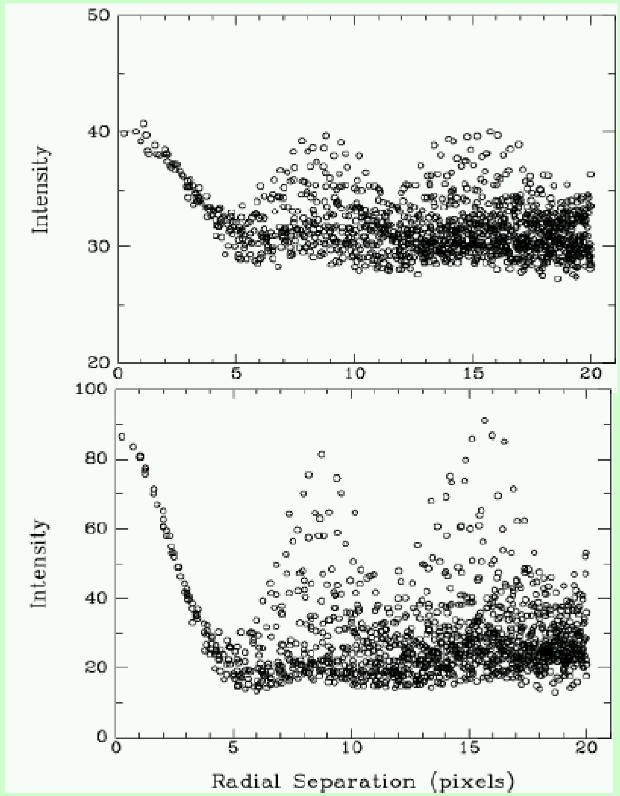
Full Bleach | 470 nm Bleach | 650 nm Bleach

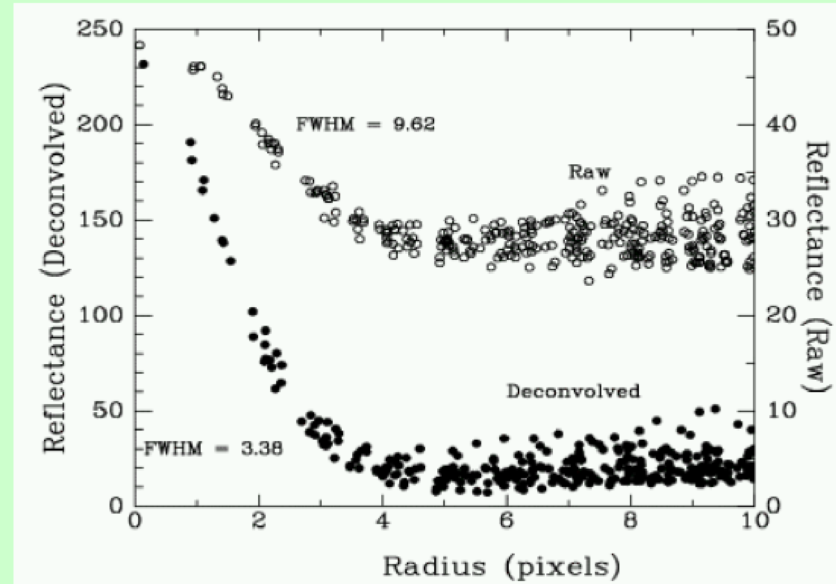


Macaque retina

Reducing source confusion with deconvolution





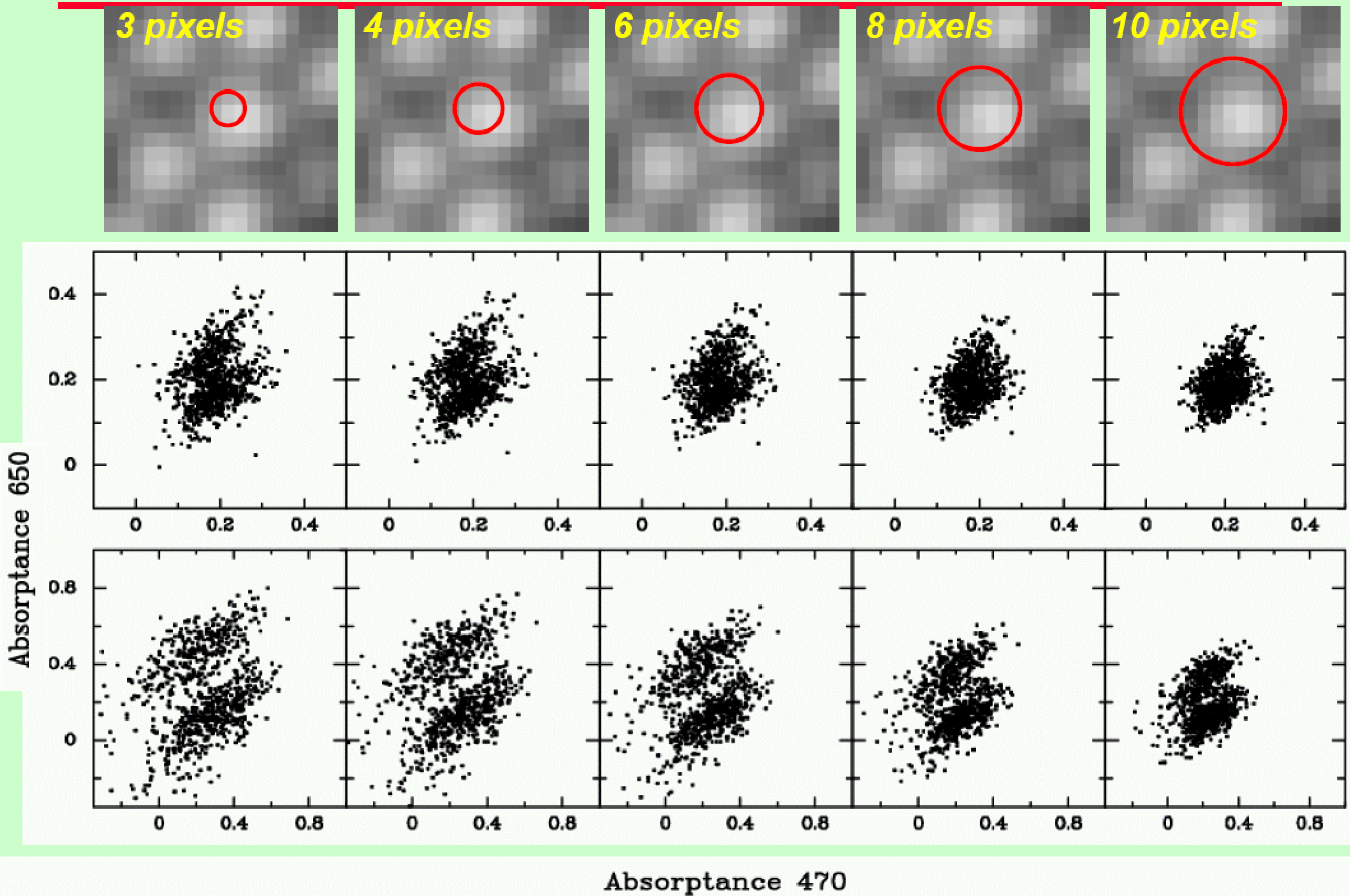


An isolated cone

Contrast Enhancement showing reduction in overlap

Macaque retina

Scatter plots of L and M cones



How to obtain the PSF?

- For the University of Rochester system, the deformable mirror is "frozen" while the retinal image is taken.
- Knowing the wavefront slopes permits a truncated Zernike model of the wavefront to be obtained (typically 65 terms with the first three, i.e. piston, tip, and tilt, set to zero).
- The modal wavefront is then numerically propagated through the "exit" pupil to obtain the focal plane PSF.
- We conducted a series of experiments imaging point sources to evaluate the quality of the reconstructed PSF.

PSF Reconstruction

Reminder:

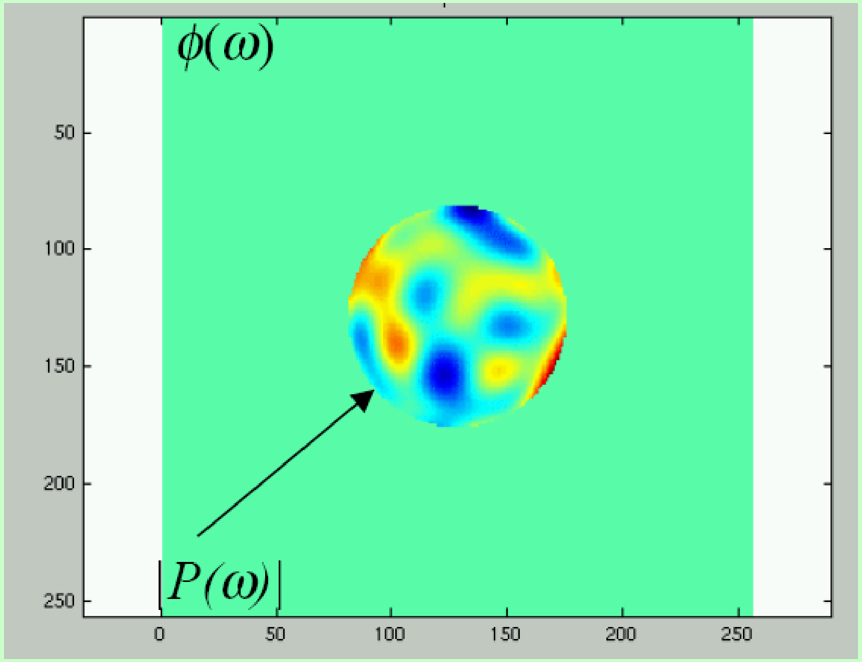
The PSF is the power-spectrum of the complex field at the pupil, i.e.

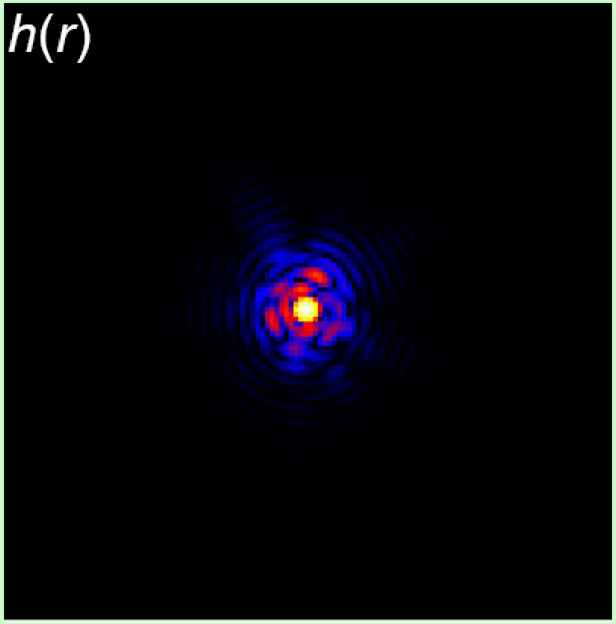
$$h(\vec{r}) = |u(\vec{r})|^2 = |\mathrm{FT}[P(\vec{\omega})]|^2$$

$$P(\vec{\omega}) = |P(\vec{\omega})| \exp[i\phi(\vec{\omega})]$$

Measure the wavefront $\phi(\omega)$ and propagate through to the image domain using the exit pupil $|P(\omega)|$ defined by unity within the pupil and zero outside.

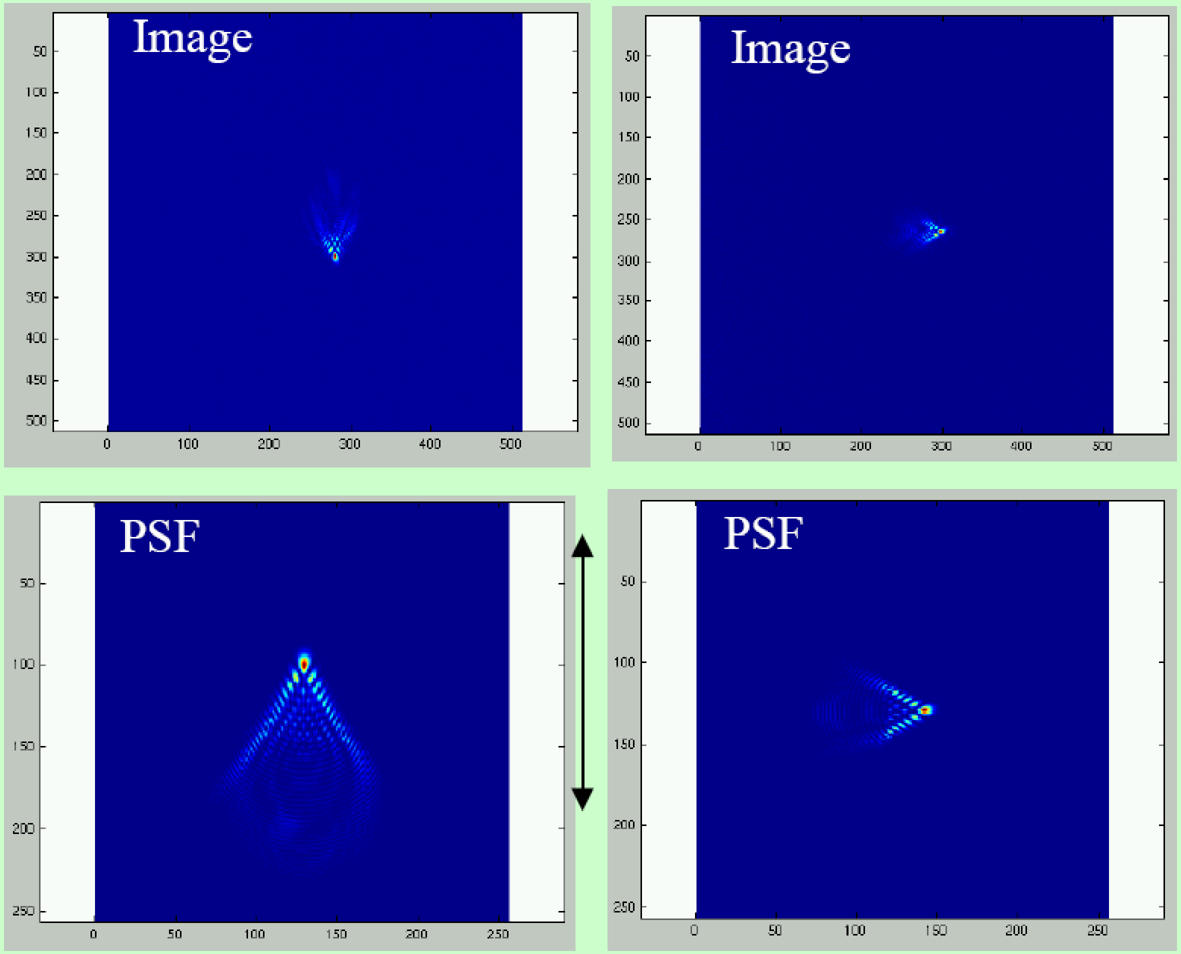
PSF Reconstruction





The Zernike modal wavefront is sampled by the "exit pupil" (left) and Fourier transformed to produce the PSF (right).

Evaluating the PSF - Orientation



Coma 1

Coma 2

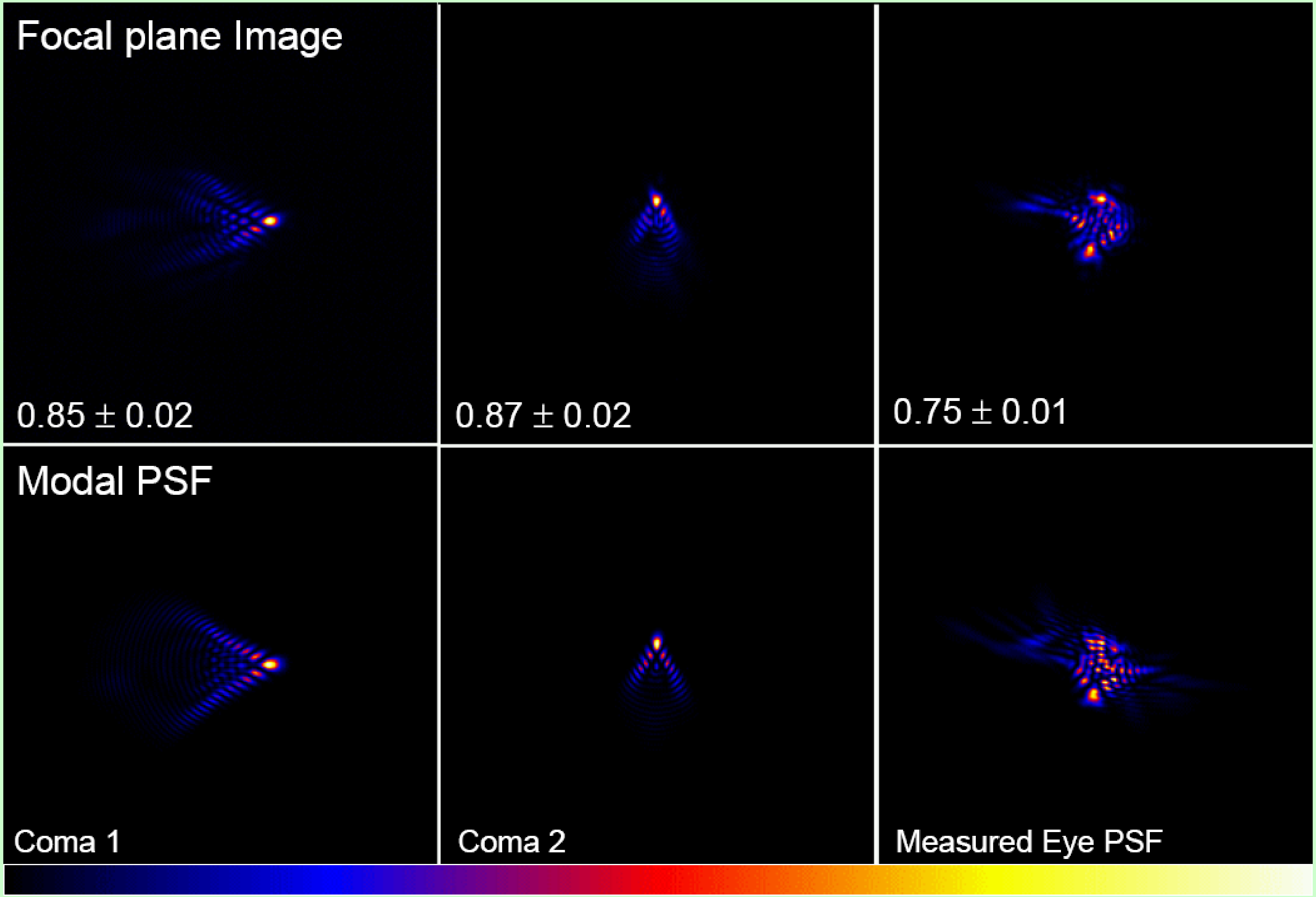
Two coma patterns were generated and applied to the reference beam producing the images on the focal plane array. (Top)

The PSFs were generated using the Zernike Coefficients.

These images demonstrate a flip about the horizontal axis, but not the vertical axis, in orientation between the two plane.

(PSFs are at half the field of the focal plane images)

Comparison of PSFs for fixed DM patterns



Deconvolution from Wavefront sensing

Multi-frame deconvolution with a "known" PSF (also called speckle holography).

The estimate of the Fourier components of the target for a series of short-exposure observations is

$$F'(f) = \frac{\langle G(f)H^*(f)\rangle}{\langle |H'(f)|^2\rangle} = F(f)\frac{\langle H(f)H'^*(f)\rangle}{\langle |H'(f)|^2\rangle}$$

Where $G(f)$ is the measured focal plane image and $|H'(f)|^2 = H'(f)\,H'^*(f)$ where $H'(f)$ is the PSF estimate obtained from the measured wavefront, i.e. the autocorrelation of the complex wavefront at the pupil.

So that $F'(f) = F(f)$ when $H'(f) = H(f)$

The Anisoplanatic Kernel

The key term in DFWS is the Anisoplanatic Kernel

$$F'(f) = F(f)\gamma(f)$$

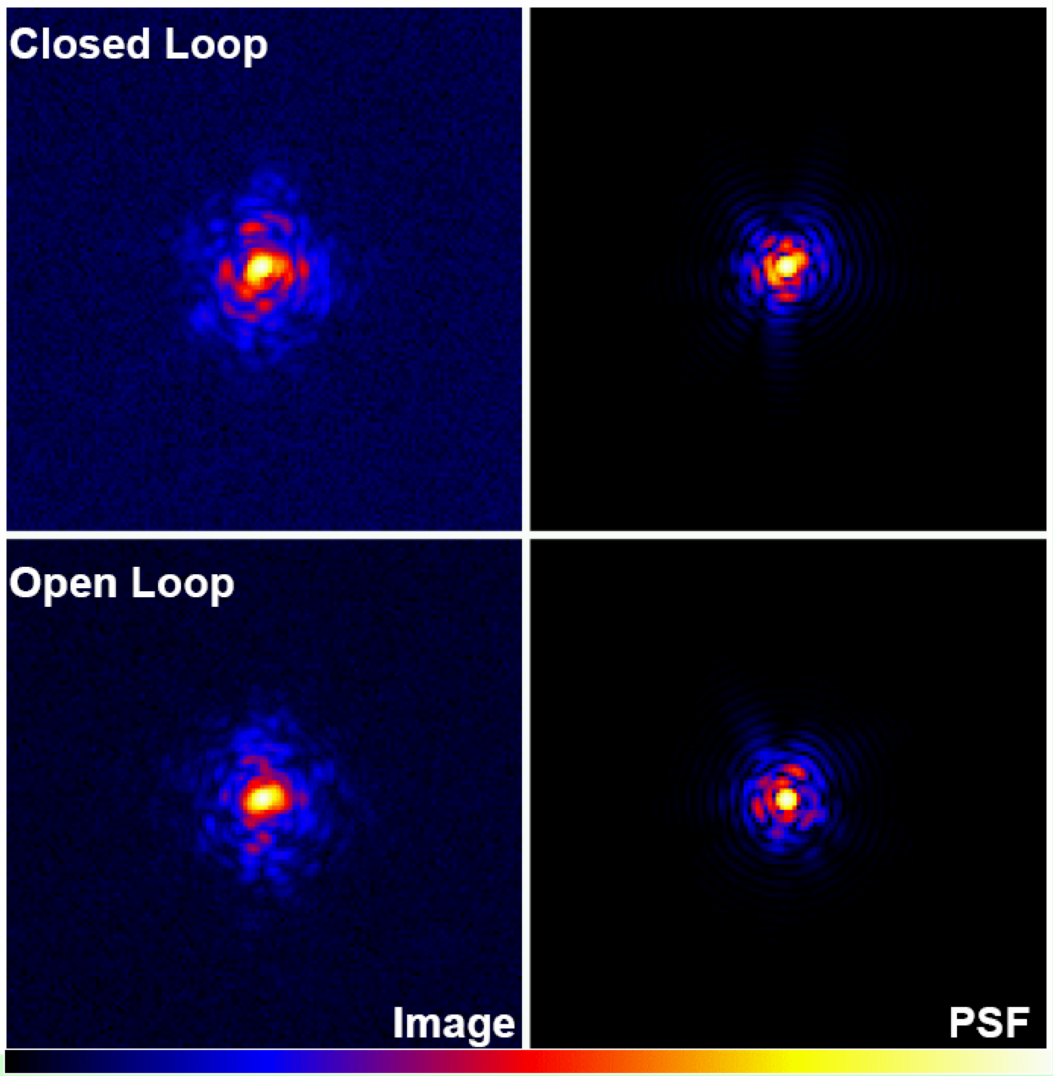
where

$$\gamma(f) = \frac{\langle H(f)H'^{*}(f)\rangle}{\langle |H'(f)|^{2}\rangle}$$

This is a normalised cross-spectrum measurement between the two sets of PSFs. When they are both equal, this term goes to unity for all measured spatial frequencies.

Measuremants of point sources through the AO system permit this to be determined.

Rochester PSF Measurements



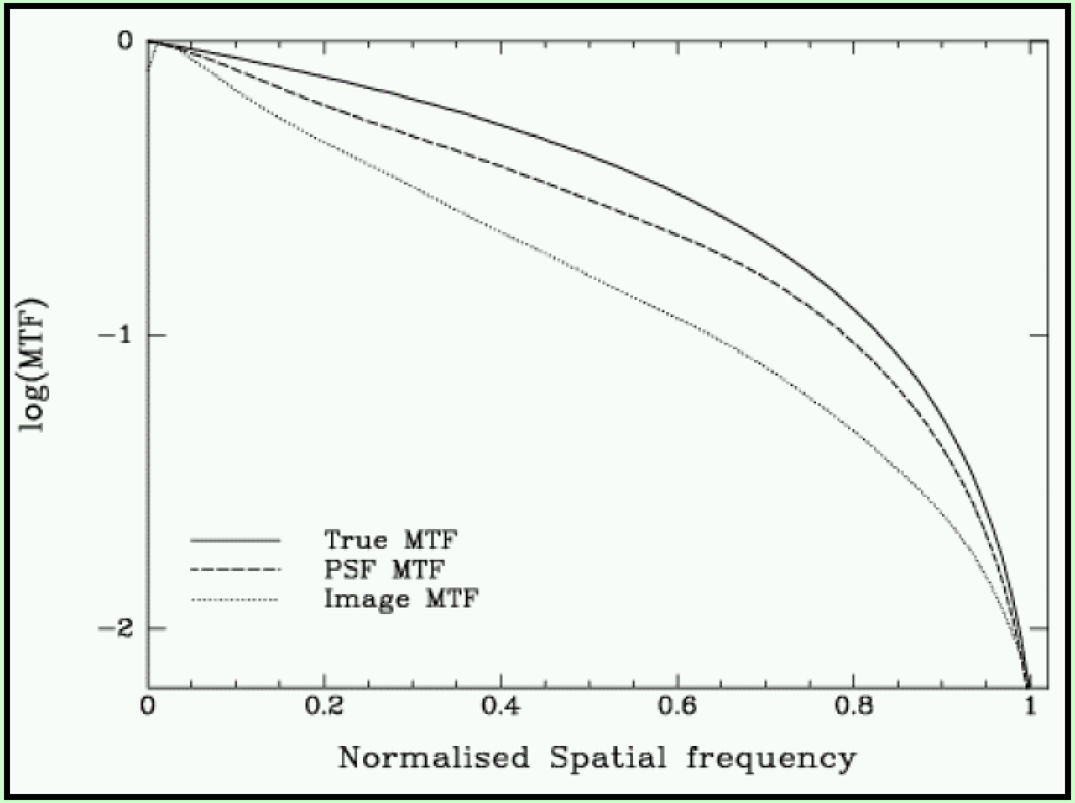
Sample open-loop and closed-loop images of the point source (laser) compared to the corresponding Zernike-derived PSFs.

Note that the focal-plane measurements look "resolved" compared to the PSFs.

The correlation coefficient for the 10-frame pairs were:

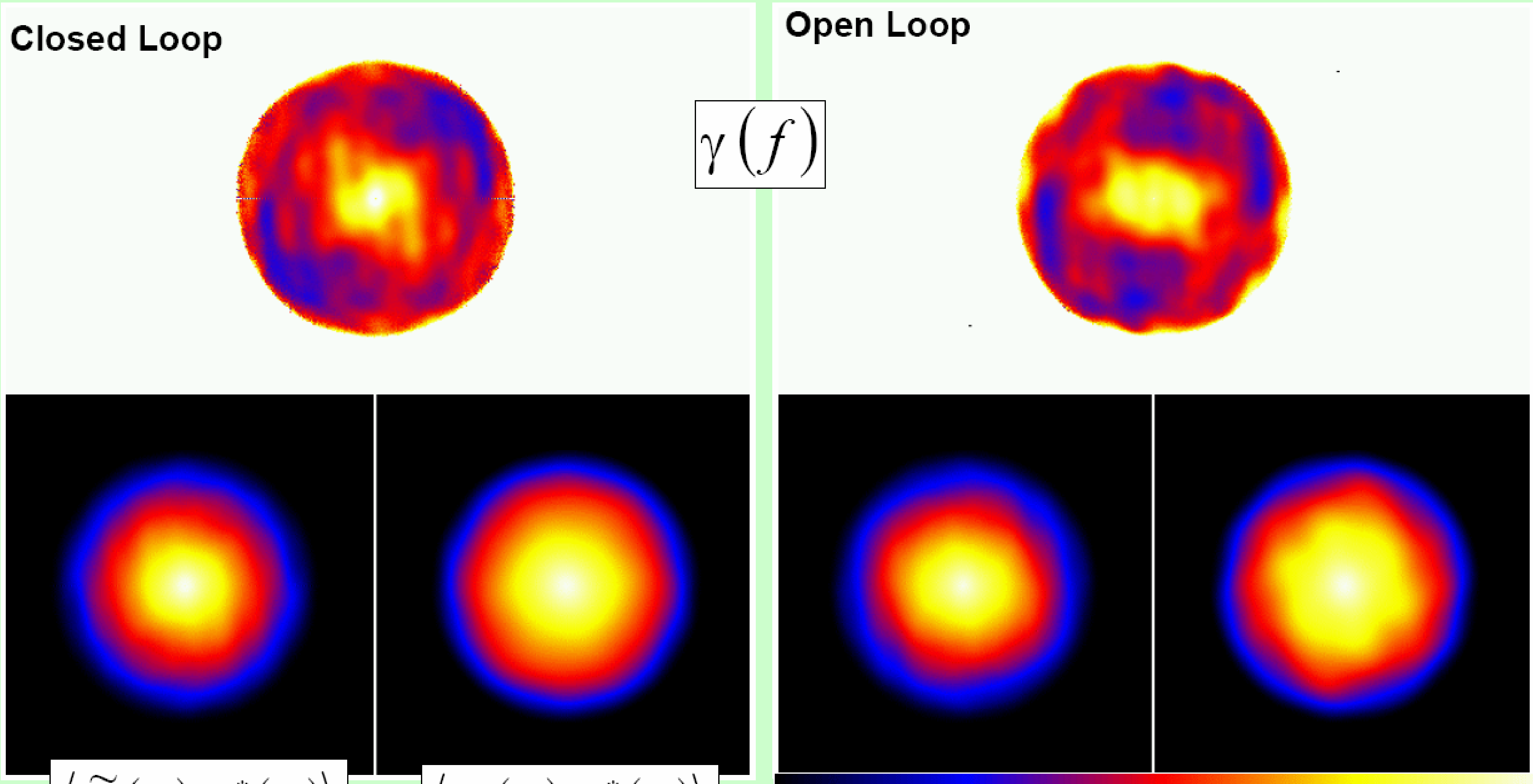
Open-Loop: 0.94 ± 0.02
Closed Loop: 0.91 ± 0.04

Rochester PSF Measurements



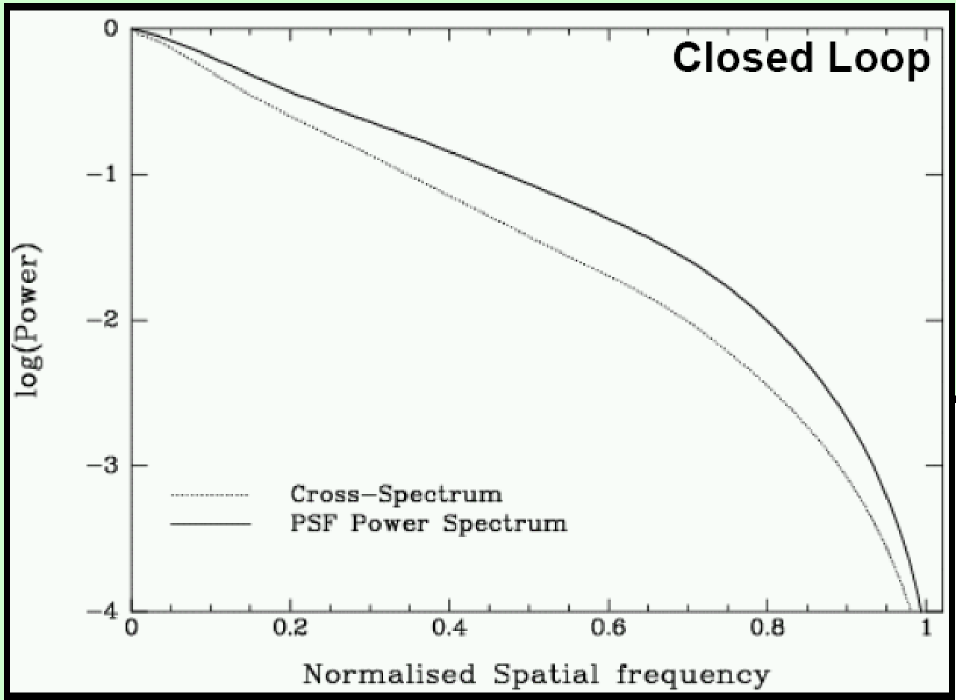
Comparison of the MTFs of the Closed Loop data to the "perfect" MTF. The image domain measurements is significantly less than the modal-reconstructed MTF.

The Anisoplanatic Parameter

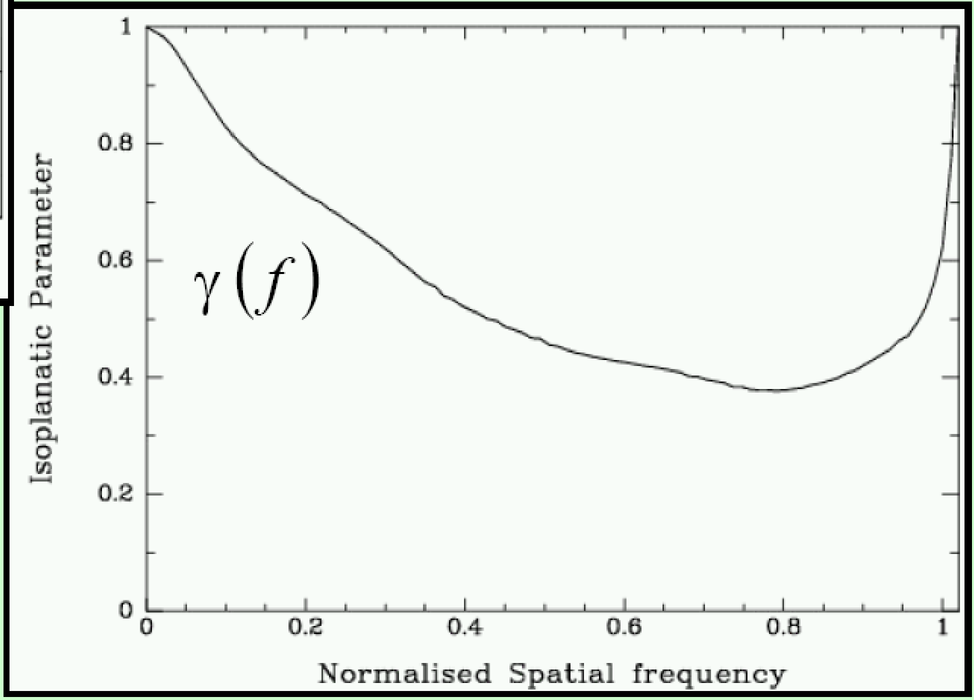


$\left\langle \widetilde{H}(f) H^{*}(f) \right\rangle$ $\left\langle H(f) H^{*}(f) \right\rangle$

The Anisoplanatic Kernel

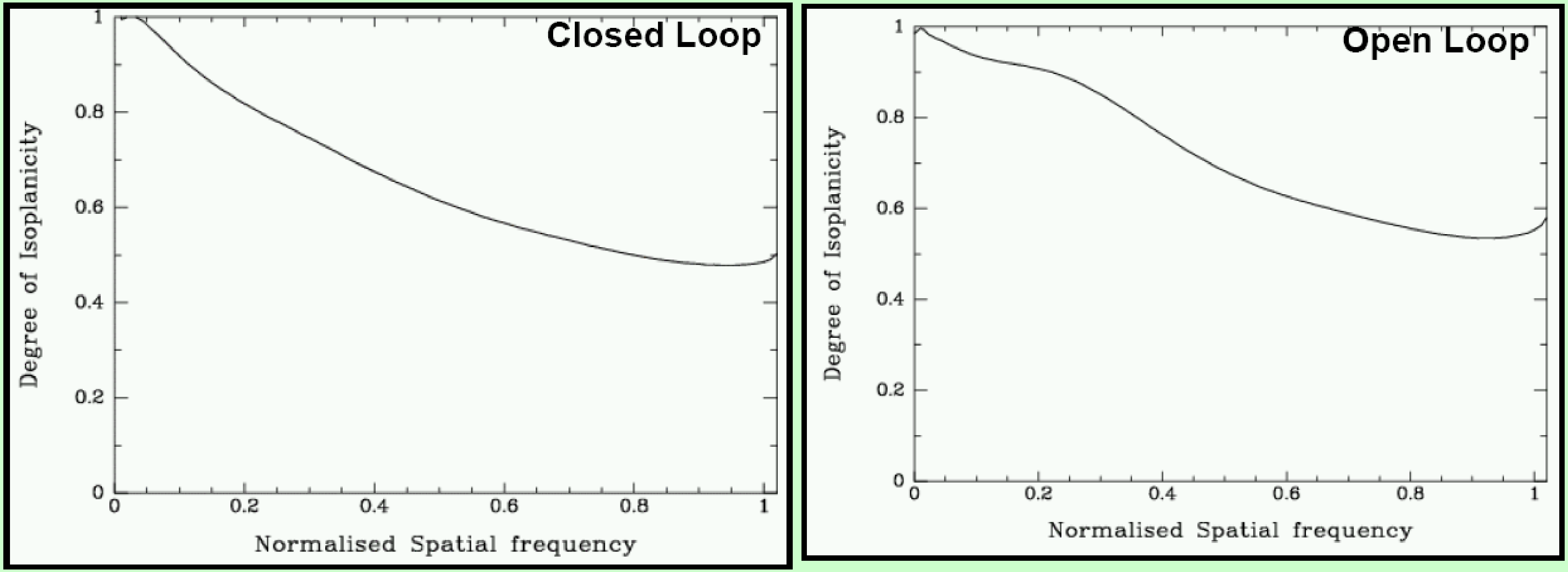


Azimuthally averaged radial profiles of the cross-spectrum, power spectrum and anisoplanatic kernel.



The isoplanatic kernel measures the correlation as a function of spatial frequency.

The Degree of Anisoplanatism



These curves suggest a significantly lower correlation between the two images than measured with a standard correlation coefficient ~ 50% at the diffraction-limit.

Why is the PSF different?

- **Modal Reconstruction as opposed to Zonal reconstruction.**

  - Are the Zernike modes missing essential wavefront information? The loop is closed on a zonally reconstructed wavefront but the PSFs are generated from a truncated modal fit.

  Need to compare a zonal reconstruction to a modal reconstruction.

- **Non-common path errors.**

  - Is there further aberration in the imaging leg not sampled by the WFS?

  If so, need to optimise the SH references for the best focal plane image, as in done for astronomical AO systems, using some sort of sharpness metric.

- **Is the PSF generated at the same spatial scale as the image?**

  - The cut-off frequencies would imply this but how to verify experimentally what the image scale is? (e.g. *Air Force target*)

- **And then there is problem of the PSF matching the corrected retinal image for the eye.**

  - tear film; eye motion …?

Adaptive Optics Performance - Sharpness

- How well does an AO System perform?
- Tools for Measurement:

  - Image Sharpness: Muller and Buffington (1975)

    S_1 or Beam Variance Metric (BVM) – Size of PSF

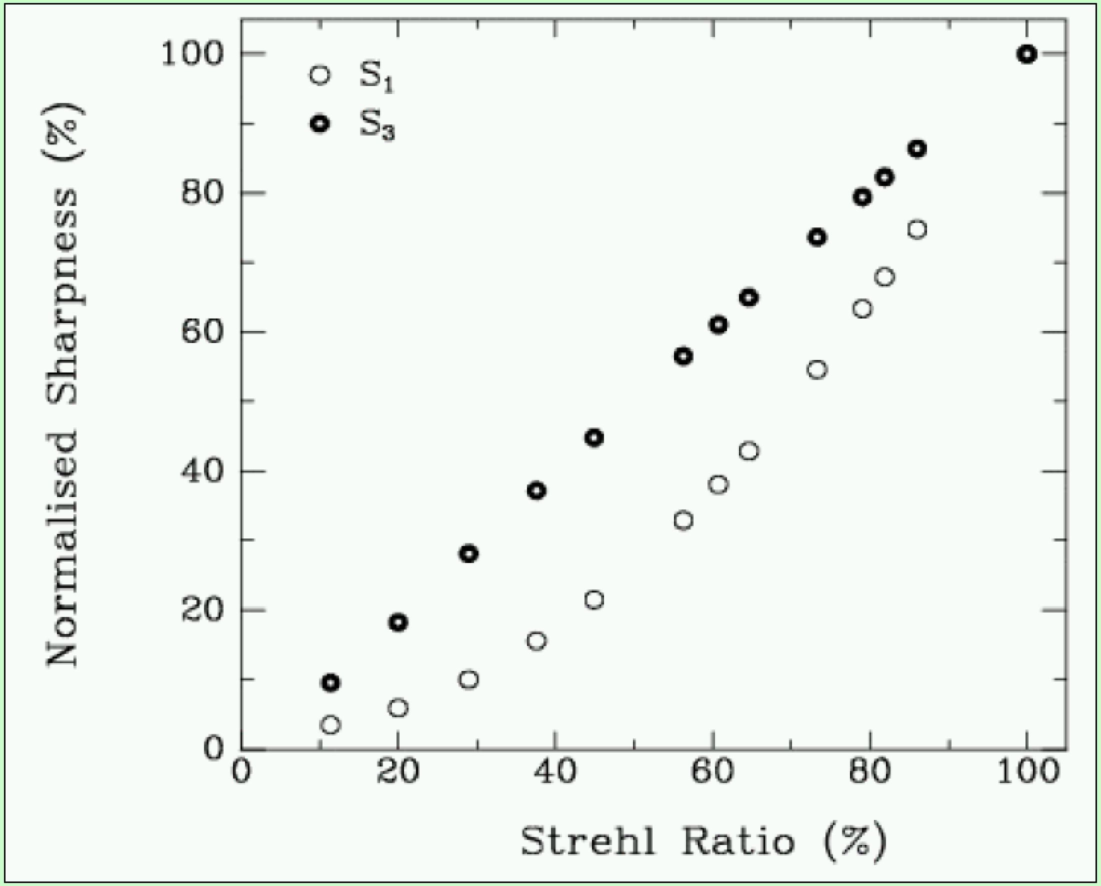
$$S_1 = \frac{\sum \tilde{h}_i^{\,2}}{\left[\sum \tilde{h}_i\right]^2}$$

S_3 or Normalized peak value – related to Strehl Ratio

$$S_3 = \frac{\tilde{h}_{\text{peak}}}{\sum \tilde{h}_i}$$

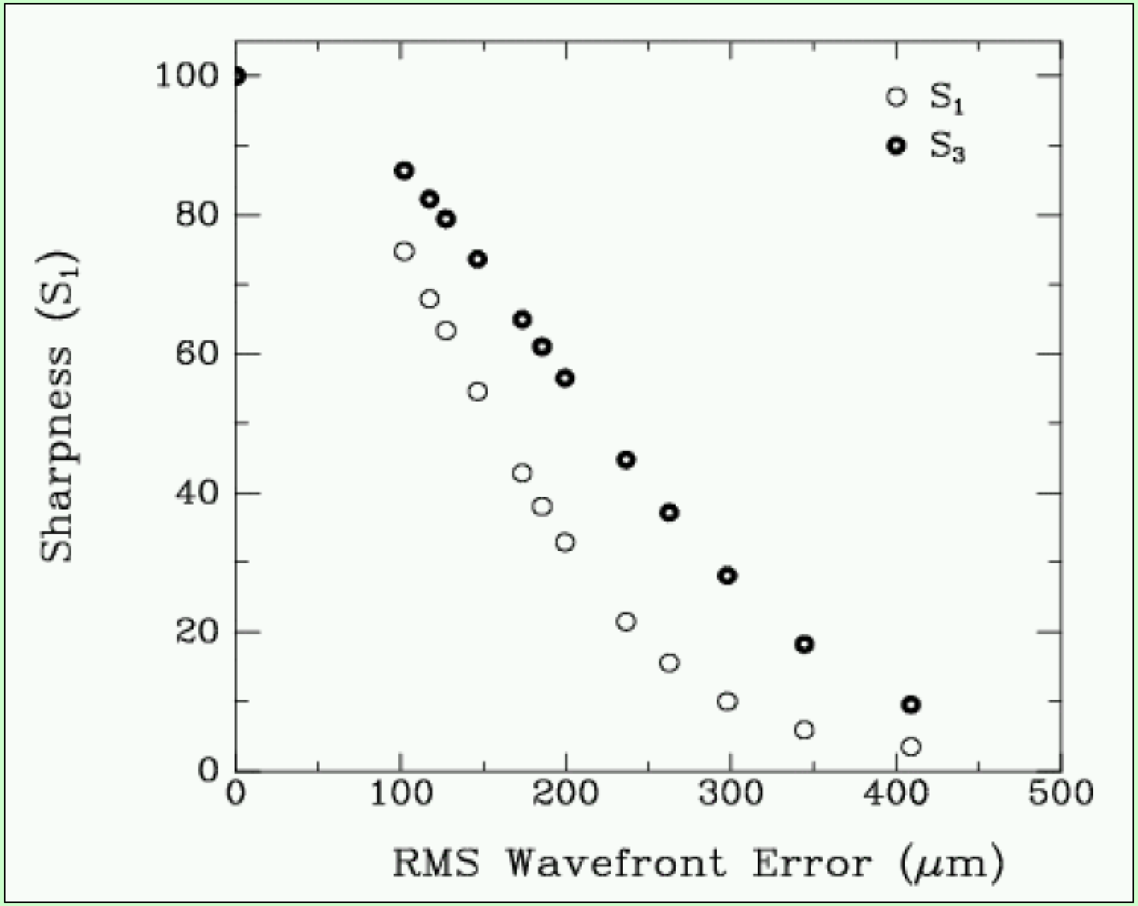
Adaptive Optics Performance - Sharpness

- Image Sharpness vs. Strehl ratio



Adaptive Optics Performance - Sharpness

- Image Sharpness vs. Wavefront Error



AO Performance Measurement

- Performance metrics
  - Sharpness and anisoplanatism measures from the AO corrected science image
  - Spectral analysis of telemetry from the AO system (wavefront sensor and deformable mirror signals)
- Astronomical AO data analysis
- Vision science AO data analysis

Credit goes to Julian Christou for developing a large portion of the methodology described in this presentation. Contact him at christou@ucolick.org

The Lick AO telemetry pipeline was developed by Don Gavel gavel@ucolick.org