

COVER SLIDE

The following slide presentation came out of discussions during the sodium workshop in Oct 2006 at Tanya Lodge, Fish Camp (Yosemite) Ca. During discussions some assumptions were made about returns from broad linewidth pump sources I'd like to comment on.



Emphasis



- At the fall 2006 laser workshop we contemplated how to estimate the return from a broad linewidth cw source, I don't think the suggestions were correct!
- These comments may explain some differences in return/watt from various sodium laser guidestar systems

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At that workshop it was suggested that the return from broad linewidth sources could be approximated from the Doppler broaden spectrum measured from a single frequency source. Some of the calculations that were suggested I do not think are the correct way of determining the expected returns.

If the measured guidestar returns from current laser systems are compared there are some differences in the efficiencies, or photon return per watt of laser power, the summary of the physics of sodium resonant scattering discussed herein suggest some of the reasons for these discrepancies.



Na Velocity Distribution \Rightarrow Doppler shift

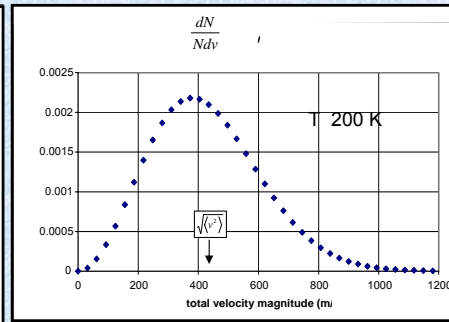
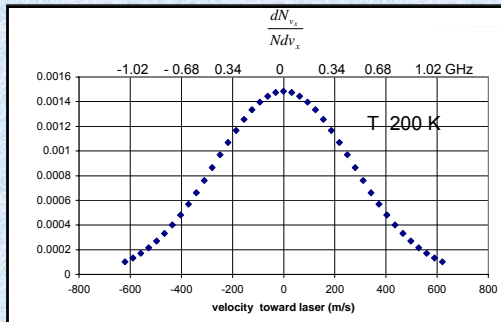


$$\frac{dN_{v_x}}{Nd v_x} = \sqrt{\frac{m}{2\pi kT}} e^{-mv_x^2/2KT}$$

$$\langle v_x \rangle = 0$$

$$\frac{dN_v}{Nd v} = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2KT}$$

$$\langle v^2 \rangle = \frac{3kT}{m}$$



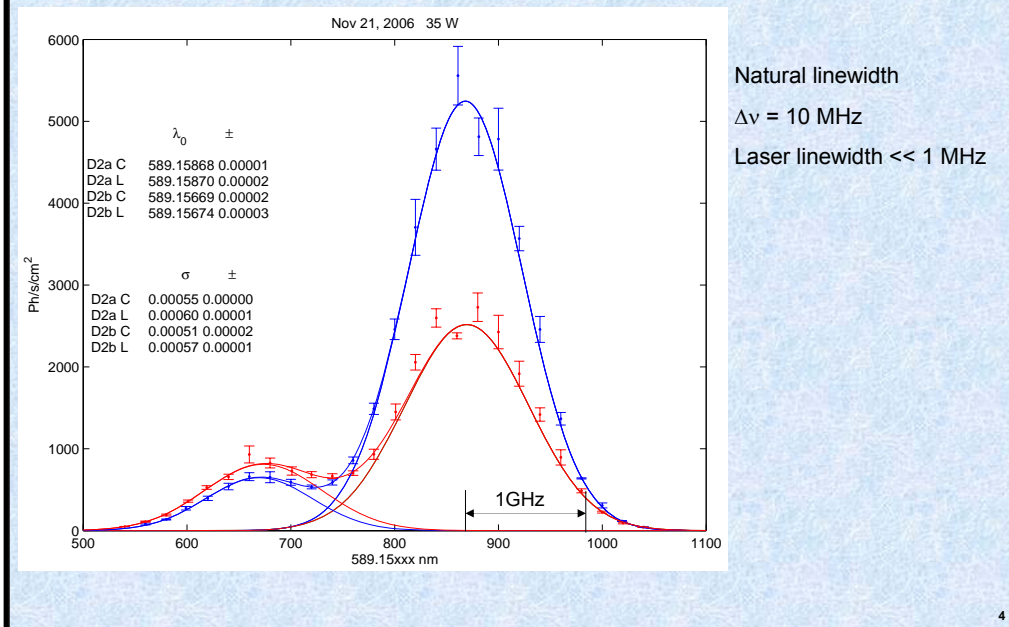
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The natural linewidth of atomic sodium is only 10 MHz, however the mesospheric sodium can be excited by a source as far as a gigahertz either side from the maximum. The cause of this is due to the distribution of velocities of individual sodium atoms, atoms with different velocities experience a different wavelength from the pump source due to the Doppler shift each experiences. The chart on the left shows the Maxwell Boltzmann distribution of velocities toward the laser source, the respective Doppler shift is shown on the top axis. Thus if the source is red shifted by 0.34GHz from line center, it would be the fraction of atoms with a velocity about 200 m/s toward the earth that would be resonant with the source. The number density at this velocity is only 12/15 ths of the maximum.

For completeness the Maxwell Boltzmann distribution of the total velocity *magnitude* is shown in the chart on the right. Although the average velocity along any given direction is zero, no atom has a velocity of zero or is standing still.



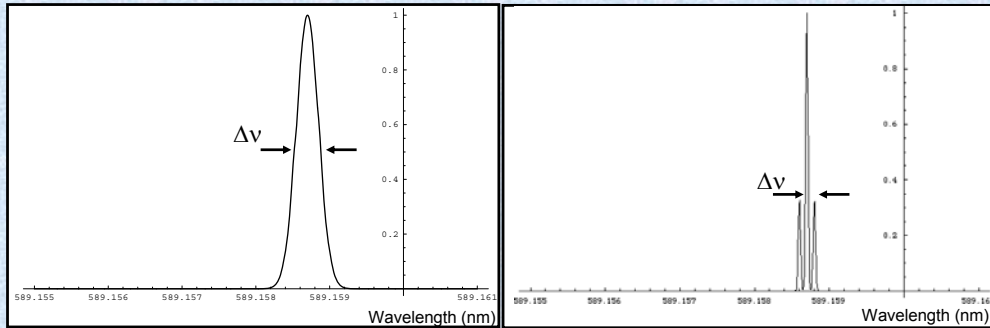
Velocity distribution leads to line broadening



The chart shows the measured return from a mesospheric sodium guidestar when excited by a single frequency source as the source is varied in wavelength across the D2 absorption line. There are two peaks due to the sodium fine structure, the smaller peak is 1.71 GHz higher in frequency and is due to the D2b transition, the larger peak is due to the D2a transition. The hyperfine transitions are not resolved due to the Doppler distribution broadening of each is much greater than their separation. The blue trace is when the source emitted circularly polarized light, the red curve when it emitted linearly polarized light. Each peak's width is due to the velocity distribution of sodium atoms, not the absorption width of individual atoms. As the source's wavelength is scanned across the D2a line, atoms of with different velocities are being excited.



Example Broad linewidth or modulated Laser Spectrum



Example of a broad linewidth source or a narrow source with a frequency jitter

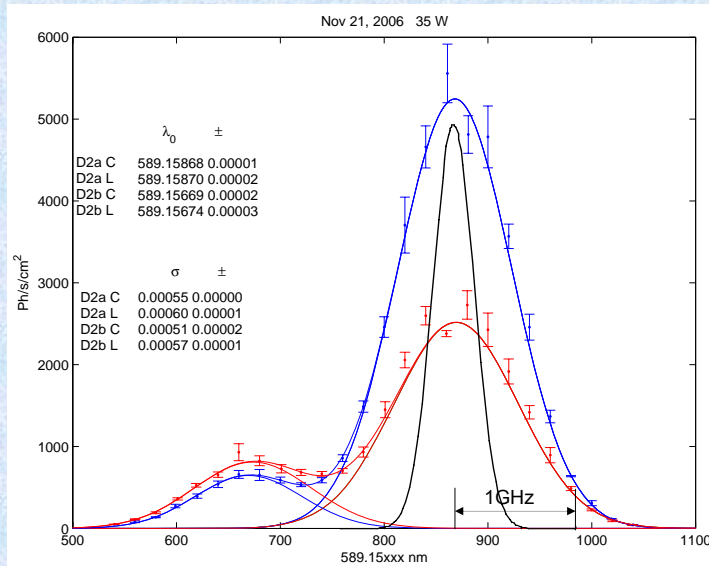
Example of a single frequency source that has been phase modulated

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The question that was raised at the last Sodium Laser workshop was “can we use the chart from the previous slide to approximate the return from a source with a linewidth much greater than 10 MHz, or a source that is phase modulated so it has one or more side bands”. This would also apply to a pulsed source or CW source with considerable jitter in wavelength where the repetition rate or the jitter has a frequency higher than either the atomic dwell time of mesospheric sodium atoms in the beam or the time it takes the sodium atoms to thermally re-equilibrate.



Return Spectrum From Single Frequency Laser and Broad Linewidth Laser Spectrum



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To gain an appreciation of the relative widths of the sources we are considering and the Doppler width of mesospheric sodium, we show them both here overlaid on one another. We are not considering sources broader than the D2a - D2b transition separation.



Suggested Return Prediction



$$\text{Return}(\lambda_0) = \int_{\text{Doppler}} \text{Return/watt}(\lambda) * \text{LaserPower}(\lambda_0 - \lambda) d\lambda$$

Return = Photon return when Laser is tuned so center wavelength is λ_0
Return/watt (λ) = return spectra from single frequency laser
LaserPower ($\lambda_0 - \lambda$) = broad linewidth source spectra/ λ , centered at λ_0

What is wrong with this?

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It was suggested that the return from such a broad linewidth source with its center tuned to λ_0 would be a convolution of the measured Doppler broadened return spectra with the source's power spectra. This estimation has also been suggested in open literature by more than one author. I don't think this is the right approach.



Must Look at Each Atom Individually Then Integrate over Doppler Profile



$$\text{Return}(\lambda_0) = \int_{\text{Doppler}} \left(\sum_{\lambda} g\left(\frac{\lambda}{1 + v_x/c}\right) \text{LaserPower}(\lambda_0 - \lambda) \right) \text{Doppler}(v_x) dv_x$$

Return = Photon return when Laser is tuned so center wavelength is λ_0

g = individual atomic response for atom at velocity v_x

Doppler = Doppler velocity profile

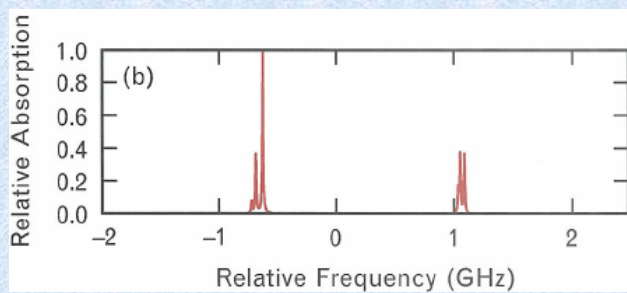
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It is suggested that the return be calculated by looking at each velocity class separately, and how it would interact with the broad linewidth source, then integrate (or sum) over all the velocity classes in the Doppler profile.

We shall now examine the response of sodium atoms to a broad linewidth source, a phase modulated source, or one with considerable jitter.



Na Absorption Spectra

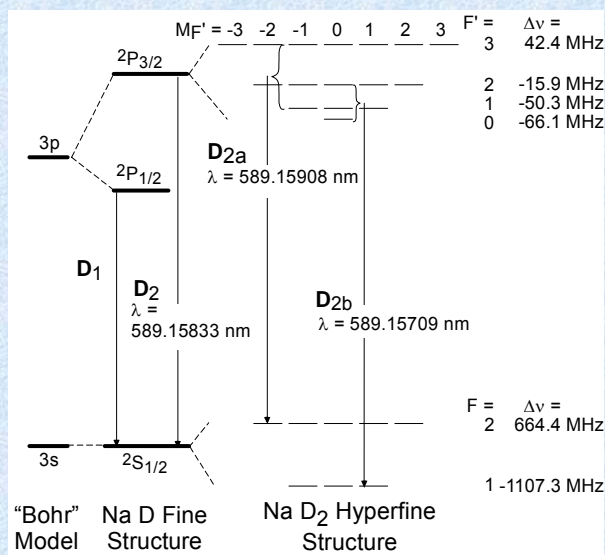


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High resolution Sodium absorption spectra. The peaks on the left are due to D2a absorption, those on the right D2b. Each of the fine structure peaks is actually resolved into three hyperfine lines.



Sodium D Energy Diagram



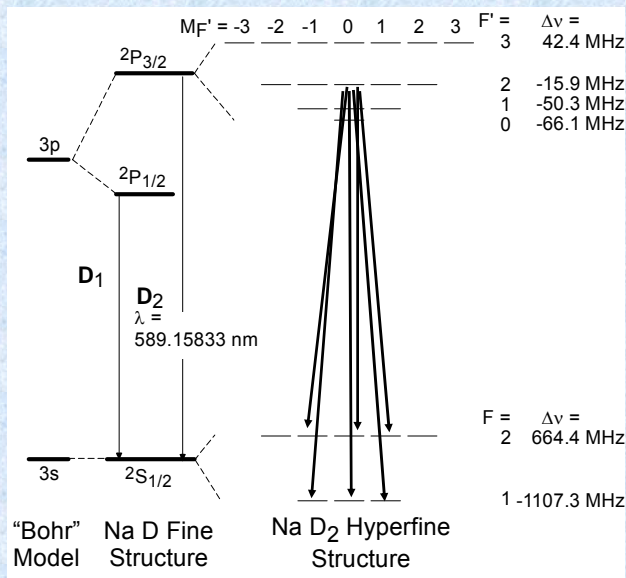
Total electron angular momentum
 $J = L + S$
 Total atom angular momentum
 $F = J + I$

All 8 ground state levels equally populated as
 $\Delta E < kT$

The sodium energy diagram showing increasingly detailed structure, starting with the Bohr model, and ending with the hyperfine structure. The fine structure is a result of the coupling between the orbital angular momentum L of the outer electron and its spin angular momentum S . The hyperfine structure is a result of the coupling of J with the nuclear angular momentum I . The atomic energy levels are shifted according to the value of F . Each of the hyperfine (F) energy levels contains $2F + 1$ magnetic sublevels, M_F , that relate to the angular distribution of the electron wave function. The hyperfine structure of $2P_{1/2}$ energy level is not shown.



Transition Rules for Spontaneous Emission



Selection Rules
for allowed
dipole
transitions

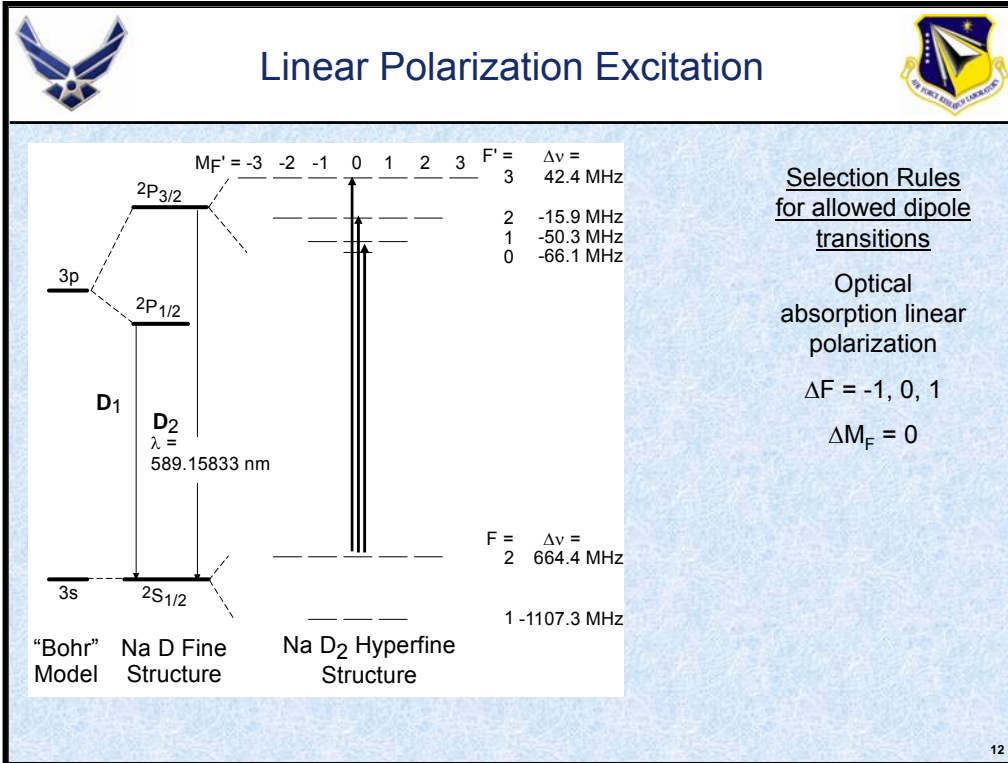
Spontaneous emission

$$\Delta F = -1, 0, 1$$


$$\Delta M_F = -1, 0, 1$$

For spontaneous emission the transition rules are $\Delta F = -1, 0, 1$


$$\Delta M_F = -1, 0, 1$$

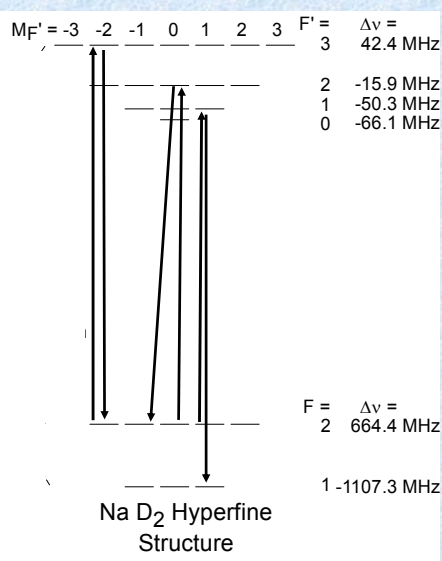


However the selection rules for excitation depend on the polarization of the excitation light. For linear polarization, $\Delta F = -1, 0, 1$, but ΔM_F is limited to 0. After many excitation/decay cycles the atoms tend to congregate to $M_F = 0$.



Steady State Linear Excitation





M_F'	F'	$\Delta v =$
3	3	42.4 MHz
2	2	-15.9 MHz
1	1	-50.3 MHz
0	0	-66.1 MHz

Na D₂ Hyperfine Structure

Steady State reached quickly

$t_L = 16 \text{ ns}$
 $t_{\text{col}} = 36 \mu\text{s}$

With single frequency laser at steady state, we are only exciting a small velocity class where $F=2 \leftrightarrow F'=3$ transition is resonant

Note those atoms at other velocities such $F=2 \leftrightarrow F'=1, 2$ transitions are resonant soon decay to $F=1$. Atoms in $F=1$ state are no longer excited.

Note $F'=3 \rightarrow F=1$ forbidden

With broad linewidth laser, atoms not only see $F=2 \leftrightarrow F'=3$, but $F=2 \leftrightarrow F'=1 \ \& \ 2$ as well, $F=2$ level becomes depleted.

Broad linewidth laser source should produce very dim guidestar (collisions repopulating $F=2$ state and winds replenishing atoms yields some photon return)

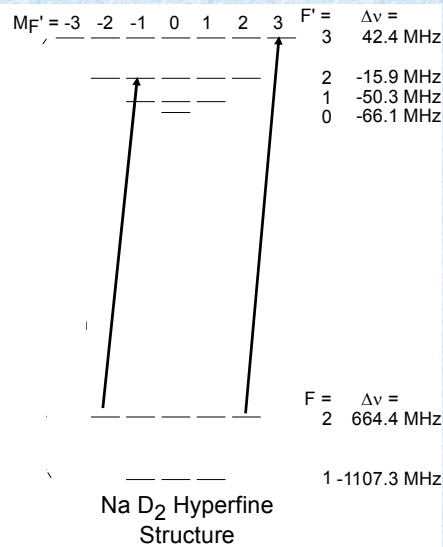
Under steady state conditions there are three phenomena to understand. With a single frequency laser, those atoms of velocities classes, where due to Doppler shifted frequency the $F=2 \Rightarrow F'=2$ or $F'=1$ transition is resonant, will absorb and re-emit. However, some will emit falling to the $F=1$ ground state, those atoms will no longer be excited. After a few milliseconds, all of these atoms will have fallen to the $F=1$ state and none of them will be excited further.

Those atoms with a velocity where the $F=2 \Rightarrow F'=3$ transition is resonant, will absorb and re-emit. These atoms can only decay to the $F=2$ ground state so they continue to absorb and re-emit. Thus, at steady state only a small fraction of the total Doppler velocity distribution are contributing to the guidestar, and its only those atoms with a velocity such that the $F=2 \Rightarrow F'=3$ transition is resonant.

However if the source has other frequency components, such that this small fraction of atoms simultaneously sees frequencies to excite atoms to the $F'=1$ or $F'=2$ state, they will *also* eventually fall to the $F=1$ ground state and will no longer be excited. Then the guidestar brightness will be limited to the ratio of these decay rates to $F=1$ and collision rethermalization and winds replenishing fresh atoms into the beam repopulating the $F=2$ ground state.



Circular Polarization Excitation



Selection Rules for allowed dipole transitions

Optical absorption circular polarization

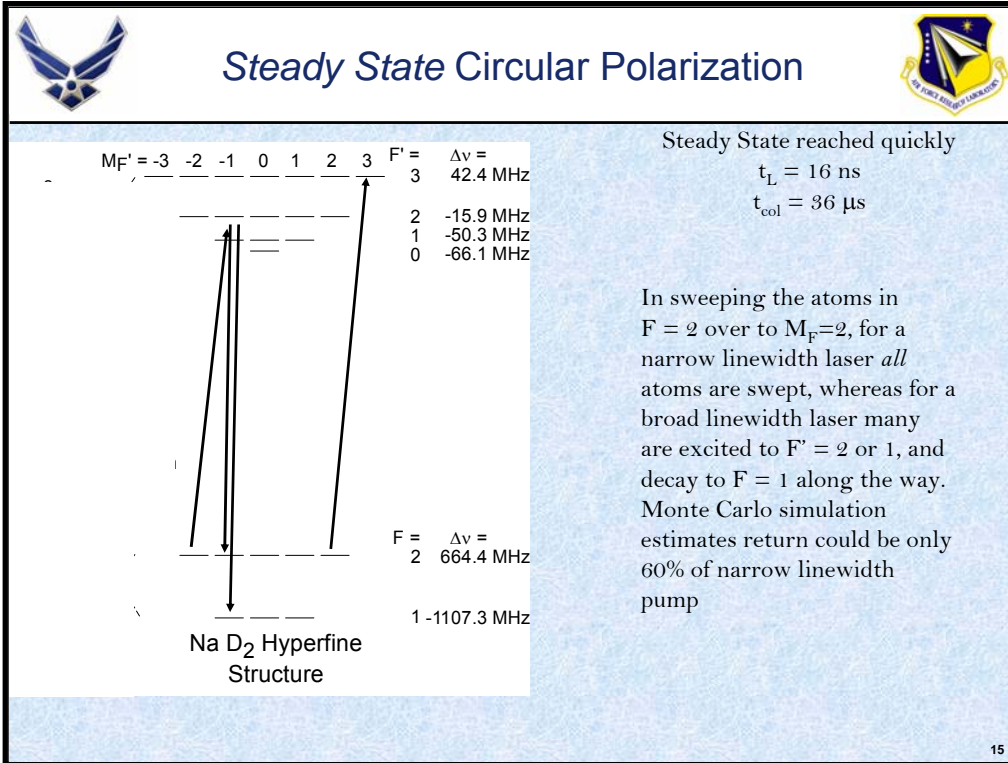
$$\Delta F = -1, 0, 1$$

$$\Delta M_F = +1$$

After many absorption - decay cycles most atoms end up in the $F=2, m_F=2$ state. This state has the largest cross section

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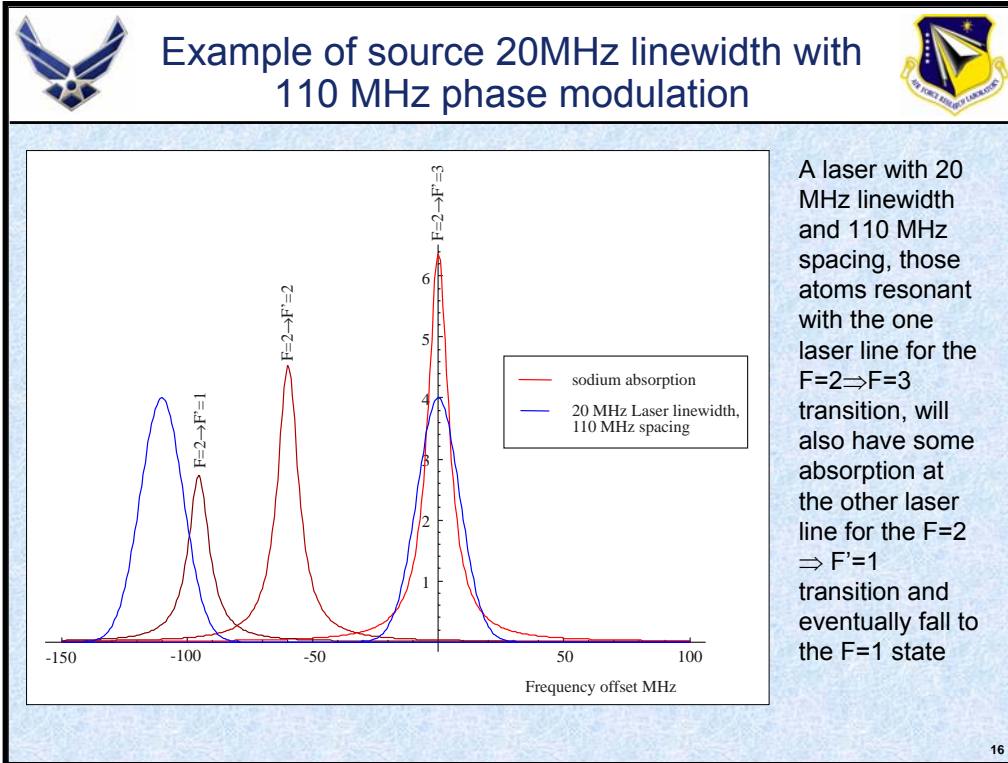
For excitation using circularly polarized light, $\Delta F = -1, 0, 1$, but ΔM_F is limited to +1 for right handed circularly polarized light pumping, and to -1 for left handed circularly polarized light pumping.



Under steady state conditions the same three phenomena mentioned for linear polarization occur with one exception. Instead of the atoms congregating in the lower $|M_F|$ states, the atoms end exclusively in either $M_F = \pm 2$. For example, with right handed polarization, and atoms of velocity such that the $F=2 \Rightarrow F'=3$ transition is resonant, for each excitation $\Delta M_F = +1$, and for each decay $\Delta M_F = +1, 0, \text{ or } -1$. After many excitation/decay cycles the atoms end up in $F=2, M_F = 2$. Once a atom ends up in $F=2, M_F = 2$, it can only be excited to $F'=3, M_{F'} = 3$, and that state can only decay to $F=2, M_F = 2$, thus it becomes a two state atom. This transition has the highest absorption cross-section and the highest directional emission back toward the source.

Again, those atoms with velocity where the $F=2 \Rightarrow F'=1, \text{ or } F'=2$ transition is resonant eventually fall to the $F=1$ ground state so at steady state conditions they contribute little return.

For broad linewidth sources, those atoms where their velocities are such that the $F=2 \Rightarrow F'=3$ transition is resonant, most of the atoms end up in the $F=2, M_F = 2$ ground state and are "optically trapped". Monte Carlo simulation with equal pumping of all three transitions show only 60% of the atoms end up optically trapped, the others fall to the $F=1$ ground state during the many absorption/emission cycles that allow the atom to change from its initial M_F state to $M_F = 2$.



For example, we will consider a source that is 20MHz wide, and is phase modulated at 110MHz. This plot shows the three $F=2 \Rightarrow F'=1, 2, 3$ transitions with Lorentzian 10 MHz FWHM line shapes, and a source line with a frequency side band from the 110 MHz phase modulation, both with a gaussian 20MHz FWHM line shape. Note the relative height of the absorption lines and the sources side band are not drawn to scale. Those atoms with a velocity such that the $F=2 \Rightarrow F'=3$ transition is resonate with the center laser frequency, the same atoms will have some overlap between the source's side band and the $F=2 \Rightarrow F'=1$ transition and after several excitation/decay cycles will eventually end up in the $F=1$ ground state and will no longer be excited. The guide star brightness will be dependant on ratio of this decay rate and the rate of thermal equilibration and winds bringing new atoms in the $F=2$ ground state into the beam.



Conclusions



- Simply convolving the laser spectrum over return spectrum from single frequency source is NOT the method to obtain total return for a broad linewidth cw laser
- CW sodium sources greater than ~40MHz linewidth will give a reduced guidestar brightness. (This is close to 5 m coherence length, narrower than typical HeNe laser)
 - For linear polarization the efficiency will be almost zero
 - For circular polarization the efficiency compared to a single frequency laser could be as low as 60%.
- This might apply to pulsed lasers, especially if repetition rate is faster than collision induced rethermalization. This also applies to sources with frequency jitter.
- These results don't apply if the linewidth $> D_{2a} - D_{2b}$ separation.
- I've modeled our returns using a single frequency source pumping D_{2a} with linear and circular light taking atomic recoil and stimulated emission into account with the sodium density being the only adjustable parameter and obtain excellent agreement. I hope to report in near future.

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In conclusion, a simple convolution of the a source's spectrum and the Doppler broaden linewidth is not the way to compute a source's efficiency in creating a sodium guidestar in the mesosphere. One must take into account how atoms in each velocity class within the Doppler profile interacts with the source, then integrate over the Doppler profile.

If the source has a spectrum where for a single atom more than one transition is resonant, the atom will eventually fall to the $F=1$ ground state and no longer be resonant.

These results apply to pulse lasers if the repetition rate is faster than the rethermalization rate and to a source with a frequency jitter. The linewidth or jitter should be considerably below 40MHz.

These results don't extend to sources with a linewidth wider than the D_{2a} , D_{2b} separation.