Parametric Design of A MEMS Actuated Nanolaminate Deformable Mirror

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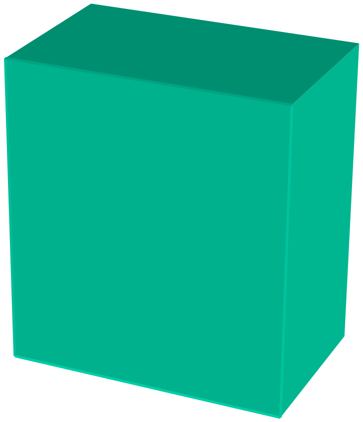
Parametric and Optimal Design: An organized way to design systems

- Come up with a conceptual architecture
- Determine goals of design
- Create a parametric model
  - Determine equations that describe important aspects of design in terms of parameters
- Use model to determine optimal design parameters

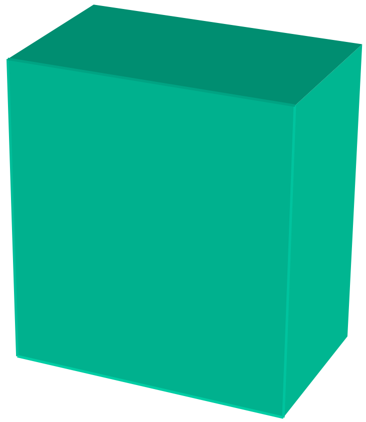
Parametric and Optimal Design Example: Define Problem

- Project: make a box to hold stuff
- Goals:
  - Volume of box must be V
  - Must use as little cardboard as possible
  - Must be rectangular



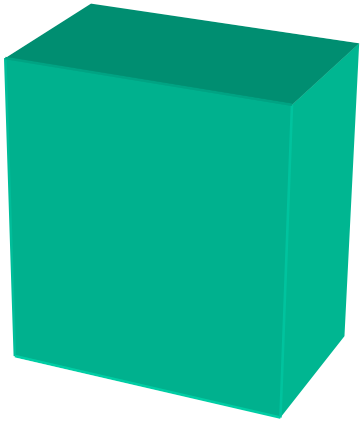
Parametric and Optimal Design Example: Determine Architecture

- Architecture: Rectangular prisim
  - All six sides made of rectangular pieces of cardboard
- Meets requirements:
  - Holds stuff
  - Rectangular



Parametric and Optimal Design Example: Determine Goals

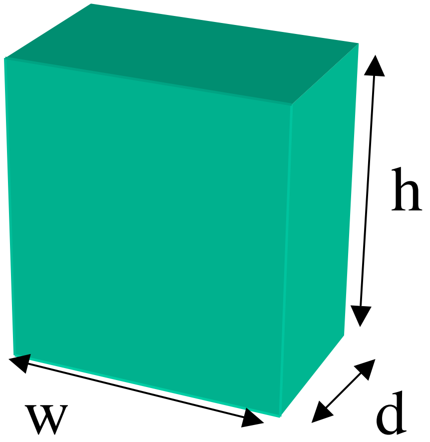
- Goals:
  - Volume of box is V
  - Surface area of box must be minimized



Parametric and Optimal Design Example: Create Parametric Model

$$V = w \times h \times d$$

$$A = 2\left(w \times h + w \times d + d \times h\right)$$

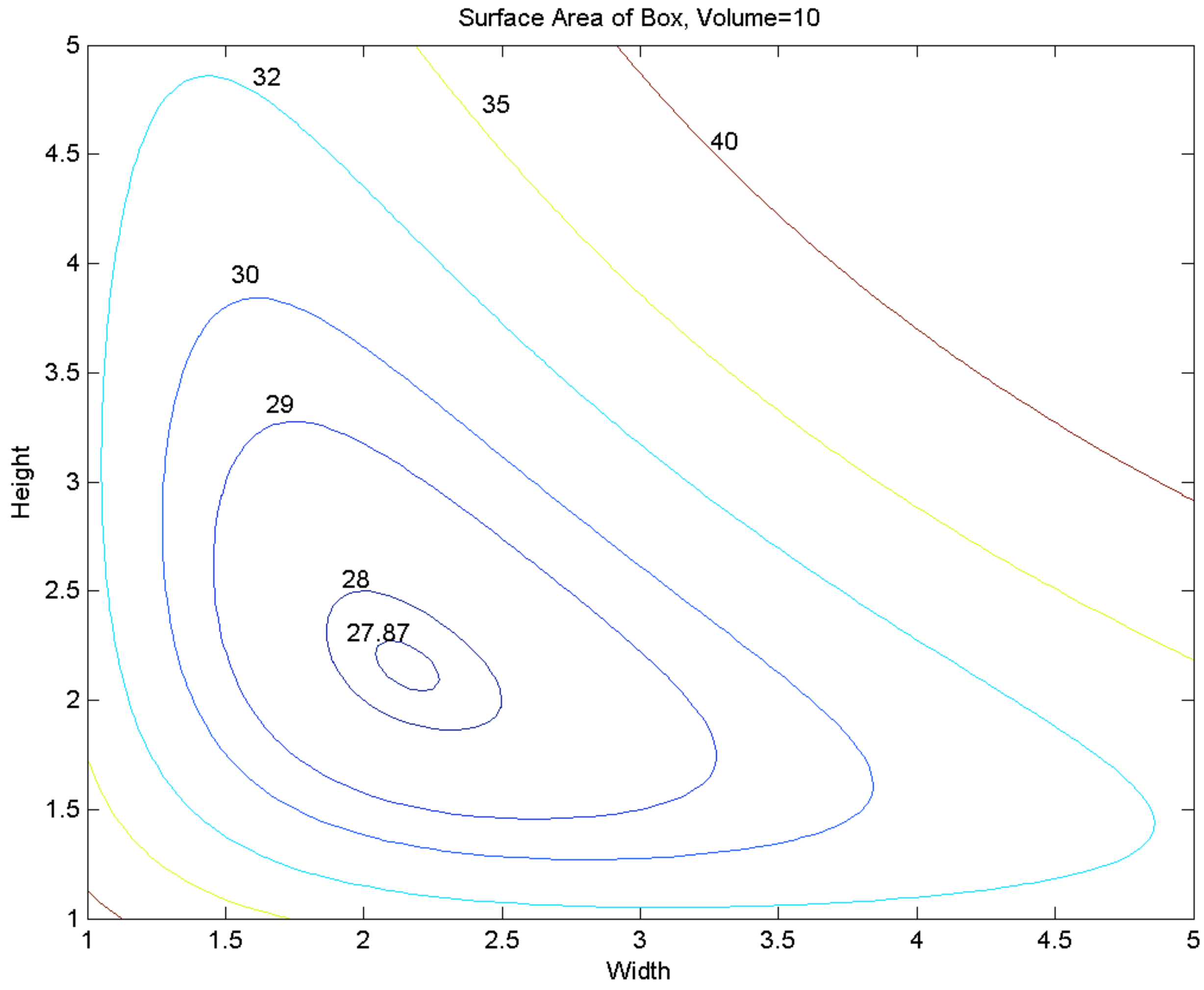


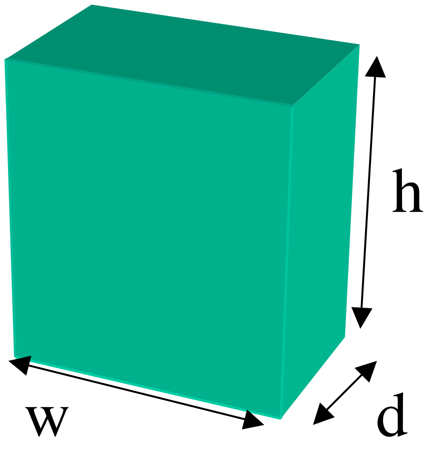
Which simplifies to:

$$A = 2\left(w \times h + \frac{V}{h} + \frac{V}{w}\right)$$

Parametric and Optimal Design Example: Use Mathematical Model to Determine Solution

Solution: $w = h = d = \sqrt[3]{V}$





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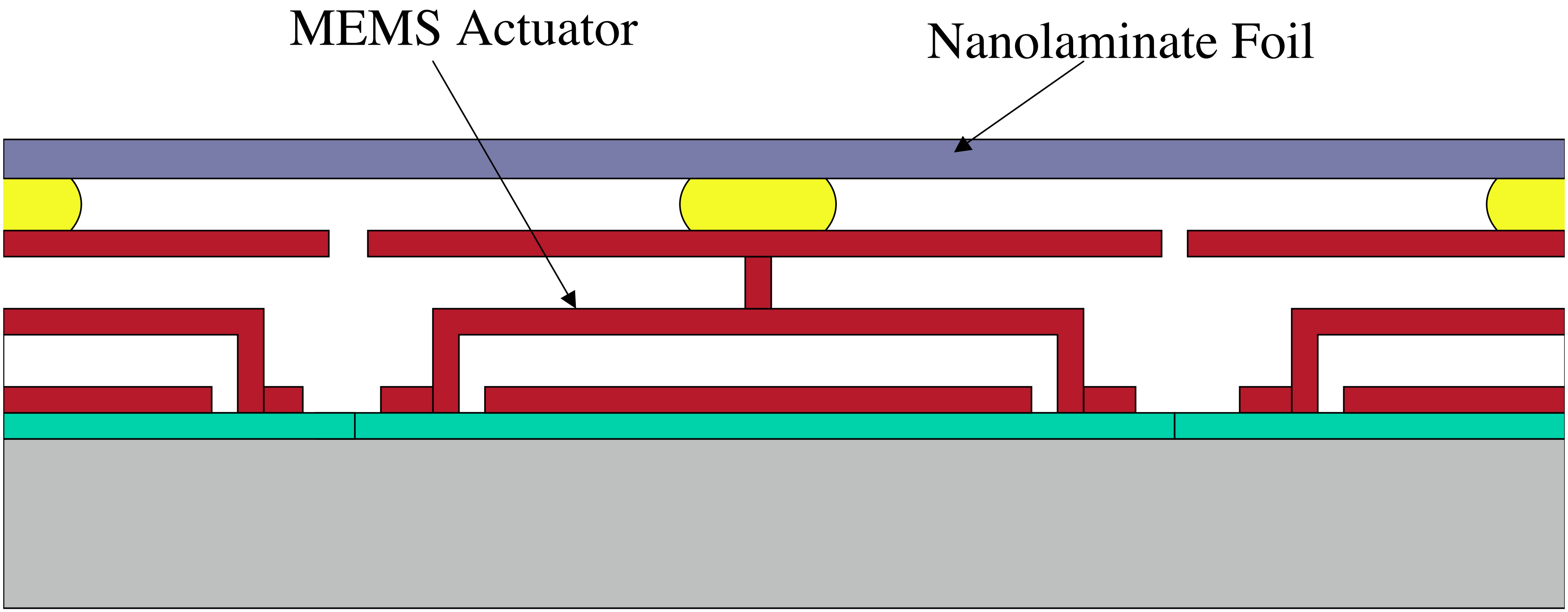
System Architecture

- Architecture
  - NL foil bonded to electrostatic actuator

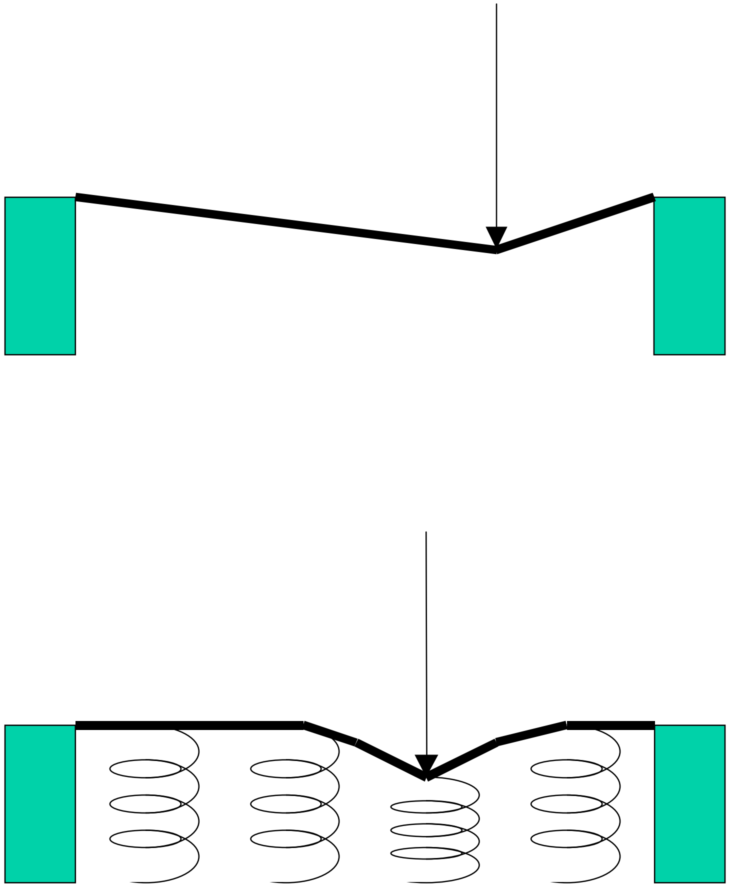


Design Goals

- Applications will specify
  - – Maximum voltage allowable
  - – Minimum natural frequency
  - – Minimum value of maximum displacement
  - – Pixel size
  - – Minimum cross-talk or minimum spatial frequency

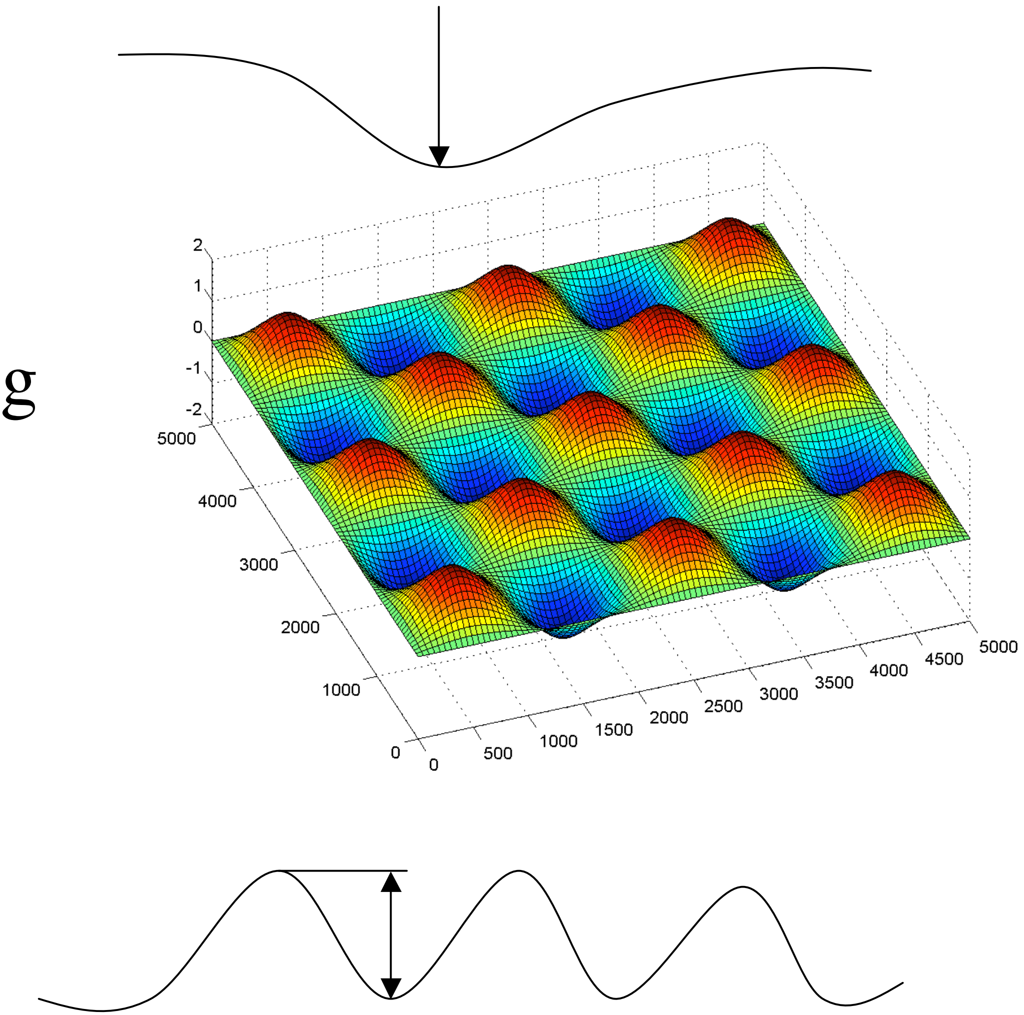
Design Goals:
Cross-talk / Spatial Frequency

- NL foil stiffer than MEMS actuator
  - Behaves like a trampoline
  - Lots of cross talk
  - Not capable of high spatial frequency features
- MEMS actuator stiffer than NL foil
  - Behaves like a mattress
  - Low Cross Talk
  - Capable of High Spatial Frequency features



Design Goals:
Checker board pattern, a rational figure of merit

- Single point load
  - Difficult to model
  - Difficult to define figure of merit
- Checkerboard loading
  - Every other pixel actuated
  - Obvious figure of merit: Difference in displacement of adjacent pixels

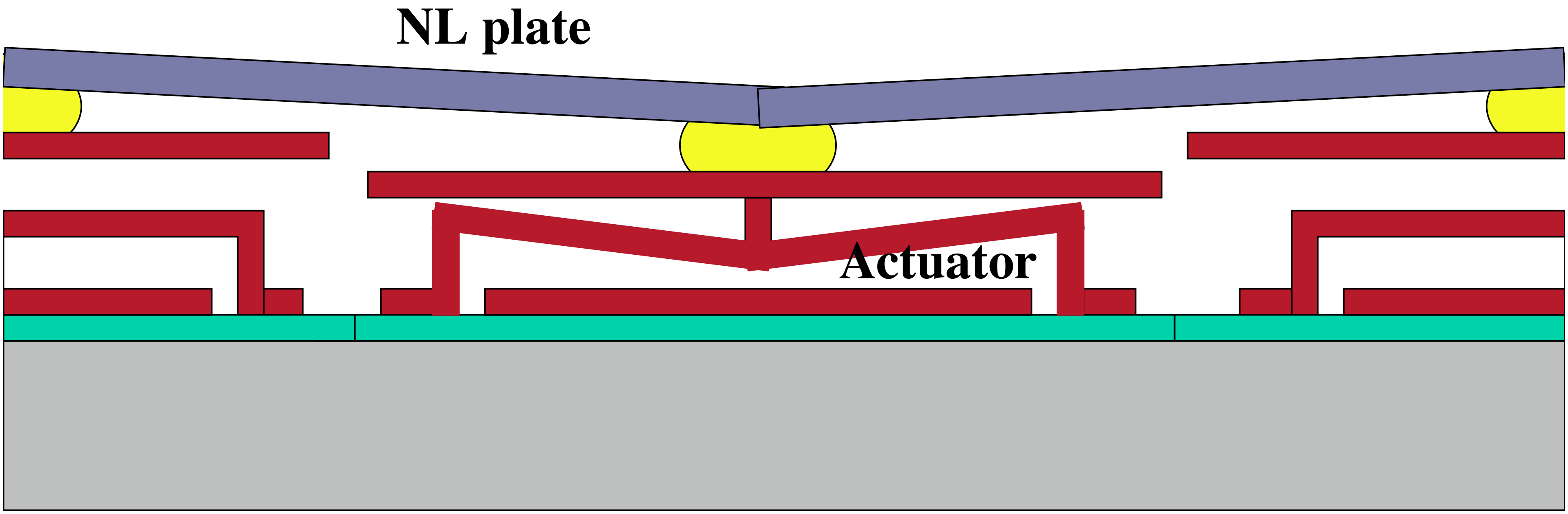


Revised Design Goals:

- Applications will specify
  - Maximum voltage allowable
  - Minimum natural frequency
  - Minimum value of maximum displacement of actuated pixel-maximum displacement of adjacent un-actuated pixel
  - Pixel size
  - Minimum cross-talk or minimum spatial frequency

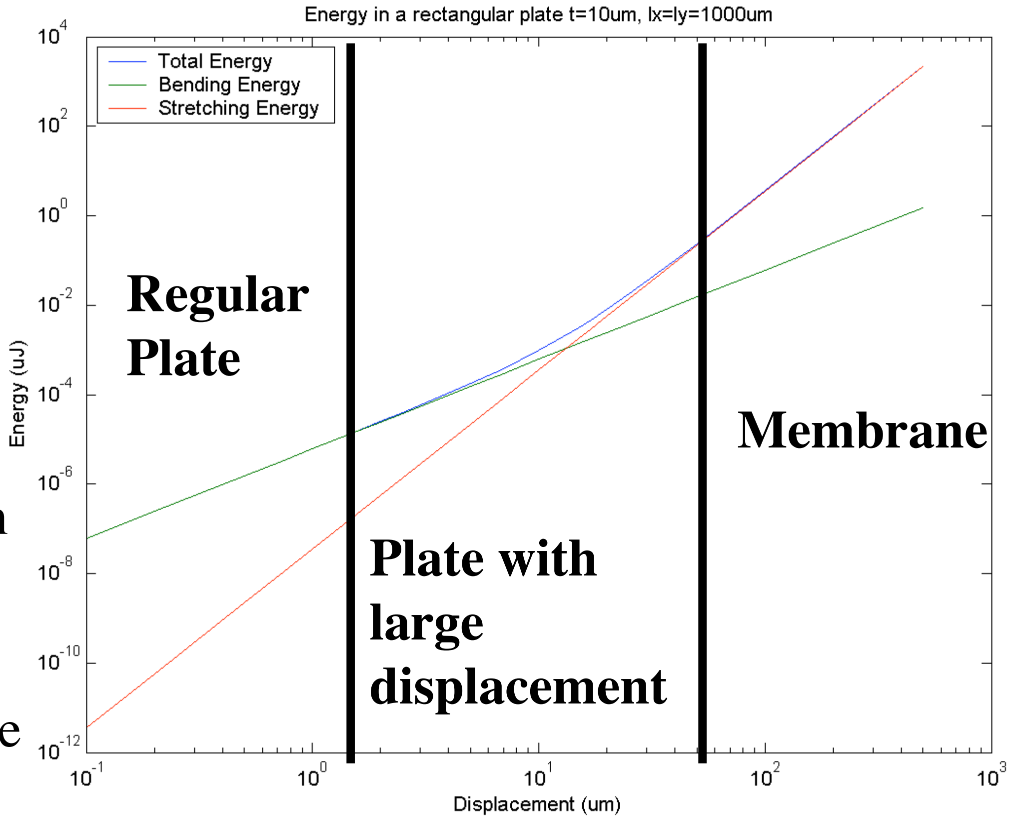
Develop Parametric Model

- Parametric Model of Actuator
- Parametric Model of NL foil
- Combine into system model



Parametric Model:
Determine Structure Type

- Regular plate
  - Resistance dominated by bending
  - Displacement less than thickness
- Membrane
  - Resistance dominated by stretching
  - Displacement much larger than thickness
- Plate with large deflections
  - Both bending and stretching are important
  - Displacement on order of thickness



Parametric Model of Actuator

Differential equation for large deflection plate

$$\nabla^4 z = \tfrac{1}{D}\left(q(x) + \tfrac{1}{2}\frac{Eh}{(1-\nu^2)}\left(\left(\left(\tfrac{\partial z}{\partial x}\right)^2 \tfrac{\partial^2 z}{\partial x^2}\right) + \left(\left(\tfrac{\partial z}{\partial y}\right)^2 \tfrac{\partial^2 z}{\partial y^2}\right)\right)\right)$$

Where

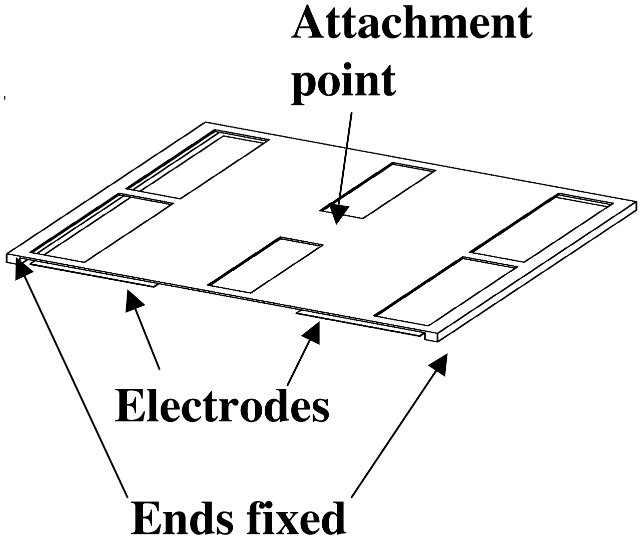
$$D = \frac{Et^3}{(1-\nu^2)} \quad q(x) = \tfrac{1}{2}\frac{\varepsilon_o V^2}{(g-z)^2}\,dx\,dy$$

BMC type actuator only bends in one dimension

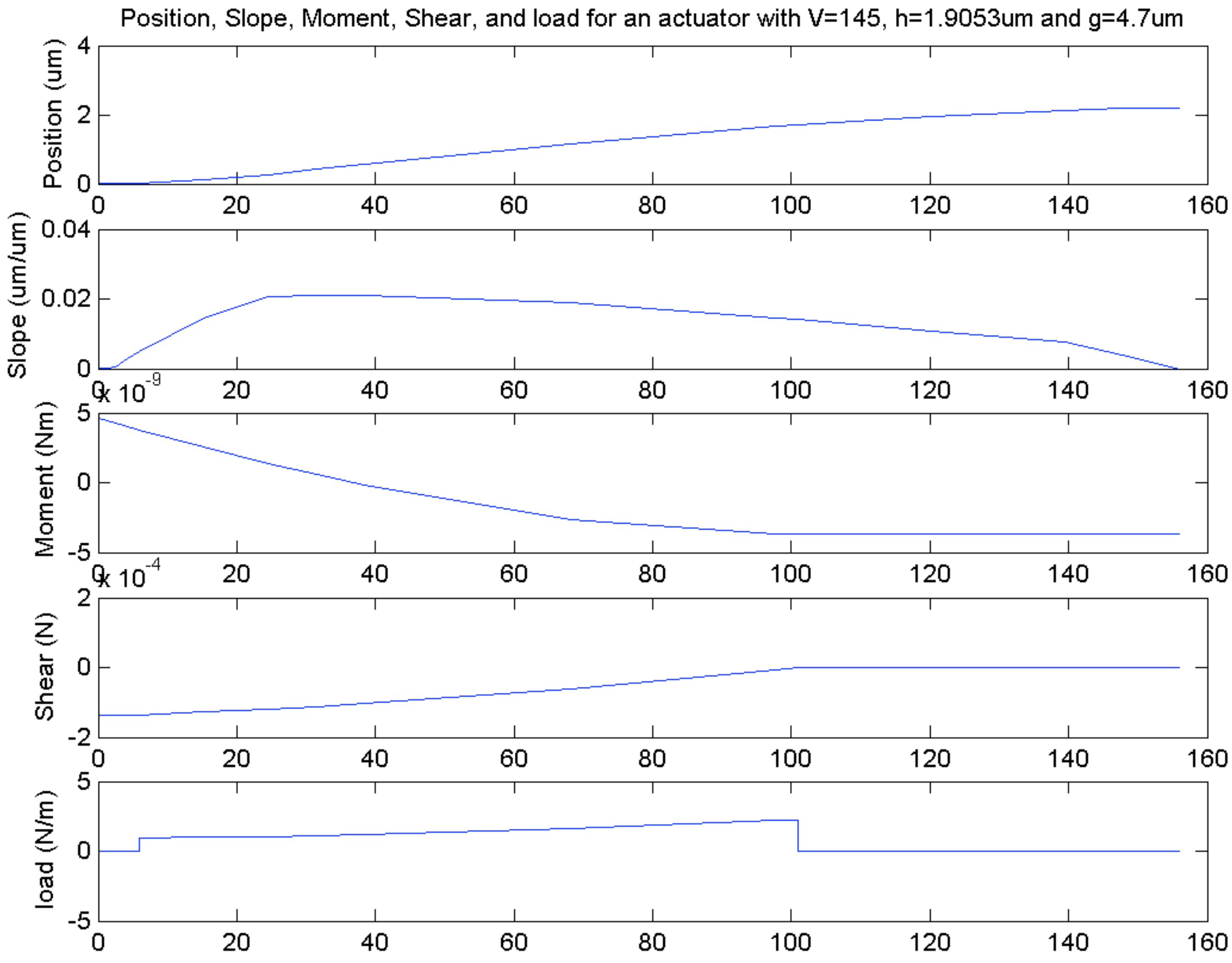
$$\nabla^4 z = \tfrac{1}{D}\left(q(x) + \tfrac{1}{2}\frac{Et}{(1-\nu^2)}\left(\left(\tfrac{\partial z}{\partial x}\right)^2 \tfrac{\partial^2 z}{\partial x^2}\right)\right)$$

Solve with ODE solver

BMC actuator

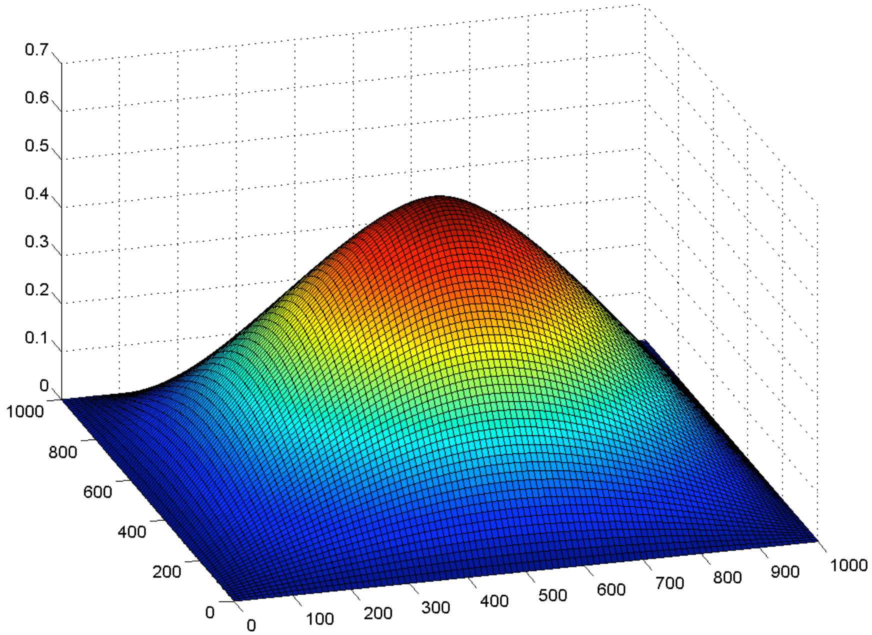


Results of Actuator Model



Parametric Model of NL foil: Solution for Plates with Large Displacements

- 2-D case of actuator problem
  - Now a PDE
- Rayleigh-Ritz Solution
  - Solve for small displacement to determine shape
  - Solve for total energy to determine large displacement behavior



Parametric Model of NL foil: Navier Solution- Regular plates

- Equation for small displacements

$$\nabla^4 z = \tfrac{1}{D}\left(q(x) + \xcancel{\tfrac{1}{2}\frac{Eh}{\left(1-\nu^2\right)}\left(\left(\frac{\partial z}{\partial x}\right)^2 \frac{\partial^2 z}{\partial x^2}\right) + \left(\left(\frac{\partial z}{\partial y}\right)^2 \frac{\partial^2 z}{\partial y^2}\right)} \right)$$

- Assume load is sum of trigonometric functions

$$q(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin\left(\frac{m\pi x}{l_x}\right)\sin\left(\frac{n\pi y}{l_y}\right)$$

  - In this case a point load for each actuator

- Displacement will also be a sum of trigonometric functions

$$z(x,y) = \frac{1}{\pi^4 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{a_{mn}}{\left(\frac{m^2}{l_x^2} + \frac{n^2}{l_y^2}\right)^2} \sin\left(\frac{m\pi x}{l_x}\right)\sin\left(\frac{n\pi y}{l_y}\right)$$

Parametric Model of NL foil:
Solve for Energy in Large Displacements

Solve for energy in plate

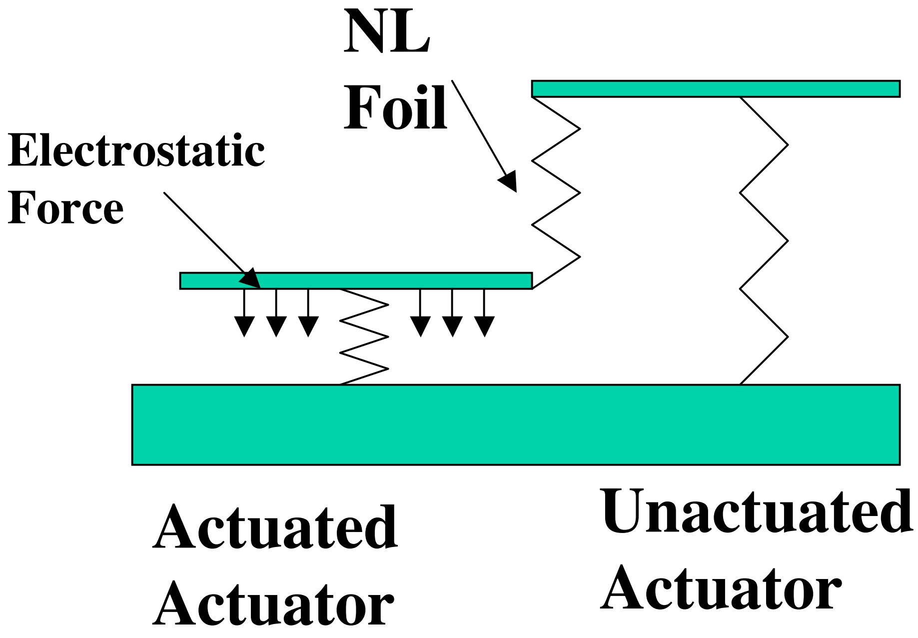
$$dU_{bending} = \tfrac{1}{2} D \left\{ \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right)^2 - 2(1-\nu) \left[\frac{\partial^2 z}{\partial x^2} \frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y} \right)^2 \right] \right\} dx\, dy$$

$$dU_{stretching} = \frac{Et}{8(1-\nu^2)} \left[\left(\frac{\partial z}{\partial x} \right)^4 + \left(\frac{\partial z}{\partial y} \right)^4 + 2\nu \left(\frac{\partial z}{\partial x} \right)^2 \left(\frac{\partial z}{\partial y} \right)^2 \right] dx\, dy$$

Use total energy to find restoring force

Parametric Model of System: Combine Actuator and NL models

- Describe DM as a system of springs

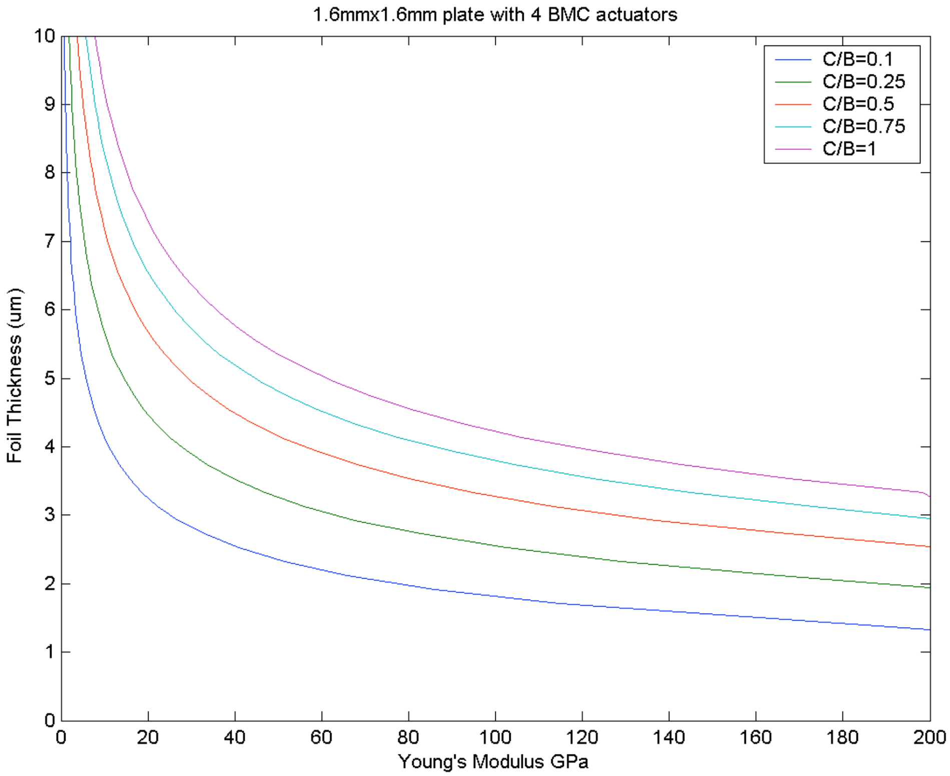


Natural Frequency of System

- Rayleigh method
  - Max potential energy equals max kinetic energy
- Solve for potential energy of both foil and actuator at max displacement
- Find kinetic energy of foil and actuator as a function of natural frequency
- Solve for natural frequency where maximum potential and kinetic energies are equal

Determine design:
Finding a NL Foil to Work With an Existing Actuator

- Actuator exists
  - All actuator parameters already defined
- Must determine NL that allows sufficient deformation



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Determine design:
Find an Optimal Actuator and Foil combination

- Determine constraints
  - Maximum and minimum dimensions
  - Maximum Voltage?
  - Max Displacement?
  - Minimum Natural frequency?
- Determine objective function
  - Voltage?
  - 1/Displacement?
- Use minimization software to find parameters that minimize objective function while preserving constraints