Open-loop Modeling of MEMS Deformable Mirror

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March 20, 2006

We present a method for open-loop control of a continuous face sheet MEMS deformable mirror of a type that is being developed by a number of MEMS manufacturers. In this type of DM, the continuous face sheet is supported by a grid of actuators which in tandem deform the mirror surface to a wavefront-correcting shape. The actuators themselves will have nonlinear response characteristics with respect to applied voltages and their own internal displacements but are otherwise single-valued (non-hysteretic).

We assume that the top face sheet can be modeled simply as a thin plate, in which case it deforms according to the plate equation:

$$D\,\nabla^4 z(x) = f(x) = \sum_{i=1\ldots n_{acts}} f_i(x - x_i) \qquad (1)$$

where $z(x)$ is the displacement of the plate, $f(x)$ is the force, and D is the flexural rigidity. The plate equation, we note, obeys linear superposition in response to forces.

At each actuator, three forces are in effect and balance to zero net force (see Figure 1): the electrostatic attraction force, $f_E(V, w)$, which depends on applied actuator voltage, V, and actuator displacement, w; the spring return force, $f_S(w)$, which depends only on the actuator displacement w; and the bending plate's response force as exerted through the supporting post, $f_P(z)$, which depends on $z(x)$ through the plate equation. The three forces balance at the post

$$f_P(z_i) = f_E(V_i, w_i) - f_S(w_i) \qquad (2)$$

where the subscript i indicates the i'th actuator.

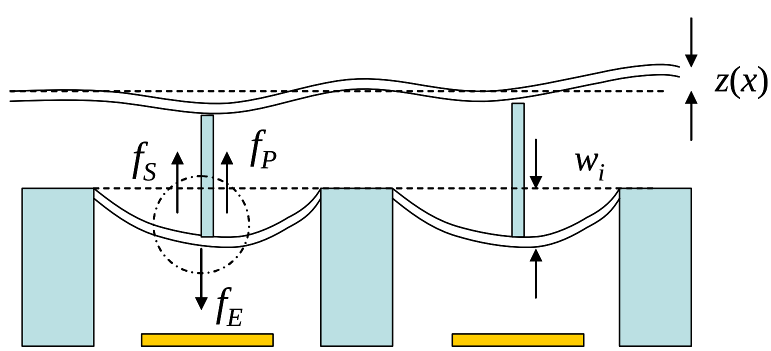

Figure 1. Forces and displacements in the MEMS Deformable mirror.

Note that a common force can be added to both the electrostatic and spring return forces to yield a net effect of zero on the right hand side of (2) and therefore not change the plate force. This situation occurs if the plate is simply translated down uniformly (piston) without changing its shape through increased electrostatic force (and compensating increased spring return force) at each actuator.

The process of open loop control of the mirror surface is comprised of two steps:

Step 1: determine the vector of actuator forces that form a given overall mirror shape

The plate force, $f_P(z_i)$, is knowable given a desired shape $z(x_i)$ through solution of the plate equation (1). Of course, only a subspace of possible shapes can be addressed due to the limitations on where the forces can be applied, i.e. at actuator locations, x_i. In essence the finite number of discrete actuator locations places a spatial frequency limitation on the controllable deformations of the surface. Given a desired shape however, it is possible to find a linear least-squares fit to the forces that best achieve this shape via the plate equation.

Step 2: at each actuator, determine the voltage that sets the desired actuator force

The next step is to find the voltages to apply to the actuators in order to achieve the derived set of plate forces.

The right hand side of equation (2) is localized to the actuator and a function of two variables, V and w, which are specific to the actuator. Since the post is assumed stiff and uncompressible, the actuator displacement, w, is equal to the plate displacement z plus a constant (the “bias”) that is common to all actuators. If we knew the functional form of the right hand side of (2), written as:

$$f_R(V_i, w_i) = f_R(V_i, z_i + b)$$

we could simply set the bias at a convenient value and, for the known plate displacement at the actuator, z_i, look up the values of V_i that causes $f_R(V_i, w_i)$ to equal the known plate force $f_P(z_i)$.

Note that this process of setting the actuator voltage (given the desired actuator force) is independent for every actuator. All of the cross-coupling in MEMS forces is through the plate force on the left hand side of (2). The right hand side of (2) denotes the actuator forces, which are not cross-coupled.

Calibration Process

The functional form of f_R can be determined empirically, through a series of tests performed on the deformable mirror in an interferometer. The advantage of the empirical approach is that it does not depend on modeling the details of the actuator structure, linearity of the spring, shape of the voltage fields, etc. Essentially our only assumptions made are that the continuous face sheet acts like a thin plate (equation (1)) and that the electrostatic and spring return forces are single valued and isolated to each actuator.

The testing puts points on a f_R vs V_i plane (Figure 2) which are each parameterized by w_i, as determined by the interferometer. Note that we must measure w, the absolute displacement of the mirror surface with respect to the supporting structure of the MEMS; knowledge of relative shape of the top surface, z, alone is not sufficient. The value of f_R is simply the (balancing) value of f_P as determined by the plate equation solution that achieves the interferometer-measured surface shape, $z(x)$. After a sufficient amount of data are collected, it is illustrative to draw contours of constant w on this graph (Figure 2) in order to see how one determines V_i given f_R and w_i.

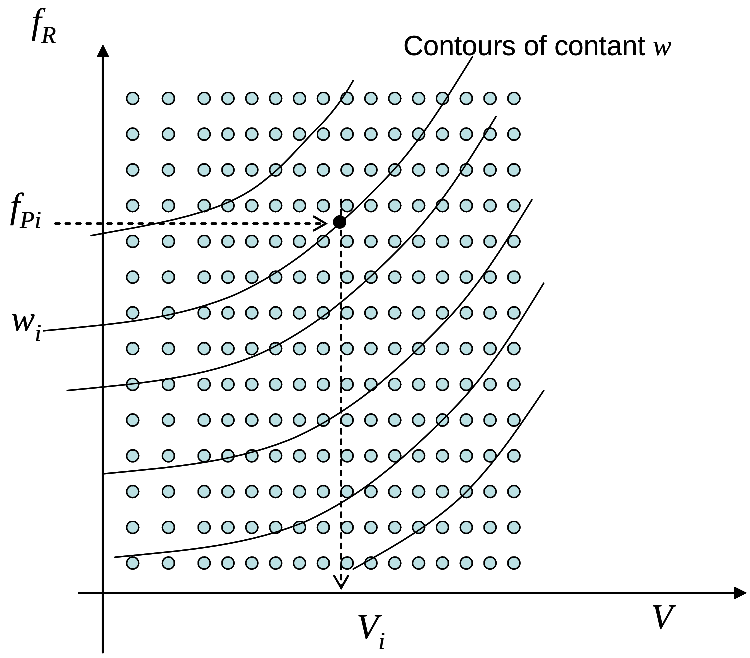

Figure 2. Graph of calibrated actuator operating points, and the lookup procedure for determining actuator voltage given plate force and displacement